

Application of Statistical Process Control in a Production Process

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Abstract: This study evaluates the process of production of Champion Breweries Plc., located at Aka Offot, Uyo, Akwa Ibom State Nigeria. The information on the following measurable characteristics used during production were obtained; Brilliance (Haze), pH, Original Gravity (O.G) and Alcohol Percentage. Information on the number of defective crates recorded for the period of fifteen (15) days on Amstel Malta were also obtained from the bottling section. Mean (\bar{X}) and Range (R) control charts for variable were adopted to ascertain if the process with respect to each quality characteristics is statistically in control. The result shows that, the out-of-control points for BRILLANCE (B) were four (4) and one (1) out of twenty (20) for the mean and range charts respectively. For pH: two (2) and one (1) out of control for mean and range charts respectively. For Original Gravity (O.G): five and zero were out of control for mean and range charts respectively. For Alcohol Percentage (A): twelve and zero were out-of-control for mean and range charts respectively. Since the out-of-control points for Alcohol have exceeded the average of all points, the entire process is disregarded, and hence the process has to be overhauled. Using the P-chart to examine the defects in the finished produce daily for 15 days; it was found that 11 points were out of control, also the need for overhaul of the entire production line. Given the overall findings it could be deduced that the process was largely out-of-control, hence the need for total overhaul and the revised control schemes as appropriate. Thus, revised control schemes were formulated for the different quality characteristics for the process to be in control and the following control schemes were proposed for the future upper and lower specifications: B (\bar{X} : 0.6563, 0.2738; R : 0.4940, 0.00), pH (\bar{X} : 3.9786, 3.7916; R : 0.2871, 0.00).

Keywords: Statistical Quality Control (SQC), Statistical Process Control (SPC), Total Quality Management (TQM), Control Charts and Control Limits

1. Introduction

Quality control being one of the prominent tools employed to ensure a certain level of quality in a product or service, has emerged as a prime tool and an important factor required by any successful industry operating in today's highly competitive business environment to incorporate in order to ensure quality of standard. Quality may be defined as conformance to requirements or expectations and need of the customer. It may also be defined as fitness to purpose. On the other hand, control is a device for operating, regulating, testing (or limiting) and keeping an activity/ process in order. Peters and Waterman (1982) found quality to be an important element in the pursuit of excellence.

Thus, quality control is simply defined as the use of

techniques and activities towards achieving, sustaining and improving the quality of products or services. It may also be defined as all the features and characteristics of the product or service that contributes to the satisfaction of customer's needs; (price, safety, availability, reliability and usability, etc.). According to Alford and Beatty (1951), quality control is an industrial management technique by means of which products of uniform and acceptable quality are manufactured or rendered. It answers the question of 'What to produce?' What is required (or expected)? When and How to carry out the inspection? And what measures to take to ensure zero defective item(s) in a production process? It can also be viewed as the technique of applying statistical methods based on theory of probability to establish quality standards and maintain it in the most economical manner. Quality control

therefore entails, integrating the following related techniques and activities such as:

- Specification of what is needed,
- Design of the product or service to meet the specification,
- Production installation to meet the full intent of specification,
- Carrying out inspection to determine conformance to the specification and to
- Make review of usage to provide information for the revision of the specifications when necessary.

On that note, every organization is therefore expected to make use of the following steps in ensuring that quality is inculcated into their process:

Setting Standards: This means determining the required cost, performance quality, safety quality, reliability quality and standard for a product.

Appraising Conformance: This involves comparing the conformance of the manufactured product or service offered to the earlier set standards.

Acting When Necessary: This involves correcting problems and their cost through the full range of maintenance of factors influencing customer's satisfaction and,

Planning for Improvements: This requires developing continuous efforts towards improving on; the costs of performance, reliability standards and the adopted safety measures.

Interestingly, solving problems related to quality is not sufficiently addressed without applying the specific tools that can help make the right quality decisions. Thus, the need for statistical tool, known as STATISTICAL QUALITY CONTROL (SQC), which helps in identifying the source of variations in the production process as well as in the finished product itself. Statistical Quality Control is the term used to describe the set of statistical tools used by quality professionals. It is a branch of quality control which involves the collection, analysis and interpretation of data for use in quality control activities. According to Montgomery (2007), the origin of Statistical Quality Control techniques could be traced to Dr. Walter A. Shewhart of Bell Telephone Laboratories in (1924), when he developed a statistical chart for the control of product variables. Later in the same decade Dodge and Rooming, both of Bell Telephone in 1939 developed the area of acceptance sampling as a substitute of 100% inspection. Shahian, et al. (1996), applied statistical quality control to cardiac surgery. He used quality control charts to analyze perioperative morbidity and mortality as well as the length of stay in 1131 non-emergent population. The result which shows that common adverse outcomes appear to follow the laws of statistical fluctuation was in statistical control. He however, concluded that statistical quality control may be a valuable method to analyze the variability of these adverse postoperative events over time. Appalasamy, et al. (2012), undertook a research on the application of statistical quality control in accessing the quality of wine production using the physio-chemical quality characteristics (Alcohol percentage, haze, original gravity, etc.).

The philosophy underlying the implementation of SQC strategy requires the company or an organization to see customers as the vital key to their company's success Shama and Kodali (2008). This implies that companies with quality control concepts rather see their corporate performance and productivity through the eyes of their customers/clients and measure them against customer/client expectation Denise et al. (2010). The predominant notion of such company therefore should not be how to make initial profit, but to give quality service to her customer(s). It should however be borne in mind that implementing quality control concept and techniques requires substantial measurement and considerable survey plan and research.

It would be interesting to note that the world economy has undergone rapid changes during the past two decades with the advent of global competition to an extent that almost every company (large or small) is touched by it in some ways. As creativity and innovations are necessary for bringing forth the change required to obtain competitive advantage, quality is the most effective factor a company or organization can be in the battle for clients/customers. To be competitive, the customers must be satisfied and to satisfy the customers, the company must focus on quality, hence Quality Control provides the philosophy and driving force for designing quality in order to delight the customers by focusing on best value of a company's products and services (Kaye and Frangou, 1998).

Although, many manufacturing companies and industries have accepted the concepts of quality control, many of such companies are yet to adequately adopt the use of statistical tools in ensuring quality of products/services due to lack of manpower, finance, etc. Most industries rather sets their goals and priorities on daily income; i. e. they consider the price of the commodity before talking about the quality and satisfying ability of the product (Xie and Goh, 1999). Brewery industries in Nigeria like many others are not left out in their inability to put in place standard measures at ensuring that required quality is built into their operation and productivity.

The question that comes to mind is; When will these companies get it right in terms of meeting their customers' needs and requirements so that the rights and privileges of these customers would not be sacrificed at the expense of the producers quest for profit maximization? Since there are a lot of competition among the brewery industries here in Nigeria, and in order to stay in the struggle, industries have no other option than to adopt and implement quality control measures in all the activities of their industries which will also give rise to the amount of profits they make.

The aim of this research is to develop appropriate quality schemes to monitor the operation of Champion Brewery Plc. To achieve the set target, the following objectives shall be adopted:

To develop variable control charts for the measurable quality characteristics used during production.

To determine if the process of production is statistically - in - control.

To build appropriate attributes control chart for the quality

of products.

To suggest an alternative control schemes for the future in event of out - of - control.

We hope this research would help to sensitize managers on the important role of quality control unit as being the engine room in any production firm.

2. Design and Methods

Champion Brewery Plc, located in Uyo, Akwa-Ibom state of Nigeria was first incorporated as a Private Limited Company on the 31st of July, 1974, and named South East Breweries Limited; but later got its name changed to Champion Brewery Plc. on the 1st of September, 1992. At the inception, the company was producing its two major products of Champion Lager Beer and Champ Malta at the initial capacity of 150,000 hectoliters per annum before an increase in the capacity to about 500,000 hectoliters per annum due to procurement of more sophisticated machinery in September 1979 and commissioning of its second production line on 11th of December 1979; the same year, when its two major products won silver medal for quality at the 16th world selection for Beers and Non-Alcoholic Beverages in Luxemburg. Meanwhile, the company's production unit is divided into two major sections, the processing section and the packaging section, with its packaging section's sub- units named "Hot Block" and "Cold Block". The Hot Block houses the Brew compartment where the Malt Plant is stocked with raw materials; and the Cold Block section is further divided into fermentation unit and filtration unit.

However, due to lack of working capital and inadequate maintenance of its plants the company was forced to shut down production between 1990 and 1991, and remained inactive until the advent of democracy in Nigeria in May 1999, when the government of Akwa Ibom state made the dreams of the reactivation of the brewery a success. Upon reactivation the company targeted at diversifying its production base with a view to bringing it to annual production of 1,000,000 hectoliters.

The data sets used in this research were collected from the Quality Assurance unit and at the packaging unit of the company under the following quality characteristics, Haze (Brilliance), Alcohol Percentage, pH, Original Gravity as well as the number of defects detected in the total daily production for fifteen (15) consecutive days. Daily readings on the four characteristics, which were randomly selected in twenty (20) rounds each, serve as the sets of data for this work. The two appropriate process control charts; the variable control charts and the attribute control charts have been adopted and briefly discussed as follows. The Mean (\bar{X}) charts and the Range (R) charts are the variable control charts while the P-charts has been chosen for the attribute control chart.

2.1. Constructing the Mean (\bar{X}) Chart

The \bar{X} - chart is based on the mean of a sample taken from the process under study. The sample contains four observations. To construct a mean chart we first need to

construct the center line of the chart. This is done by calculating the means of each sample. The center line of the chart is then computed as the mean of all k sample means, where k represents the number of samples;

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_k}{k} = CL \quad (2.1)$$

To construct the upper and lower control limits of the chart, we use the following formulas:

$$\text{Upper control limit (UCL)} = \bar{\bar{X}} + Z\sigma_{\bar{X}} \quad (2.2)$$

$$\text{Lower control limit (LCL)} = \bar{\bar{X}} - Z\sigma_{\bar{X}} \quad (2.3)$$

Where; $\bar{\bar{X}}$ = the average of the sample means.

Z = Standard normal variable (2 for 95.44% confidence, 3 for 99.74% confidence).

$\sigma_{\bar{X}}$ = Standard deviation of the distribution of sample means, computed as: σ/\sqrt{n} .

σ = Population (process) standard deviation.

n = Sample size (number of observations per sample).

Alternatively;

The control limits can be constructed by using the sample range as an estimate of the variability of the process. The control limits for the range, R (max-value – min-value in the sample) is given as:

$$UCL = \bar{\bar{X}} + A_2\bar{R} \quad (2.4)$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} \quad (2.5)$$

Where; $\bar{\bar{X}}$ = Average of the sample means.

\bar{R} = Average range of the samples; $\bar{R} = \frac{\sum_{i=1}^k \bar{R}}{k}$.

A_2 = Factor obtained from the table (see appendix).

NB: A_2 is a factor that includes three standard deviations of ranges and is dependent on the sample size being considered.

2.2. Constructing the Range (R) Chart

The R-Chart is constructed in a manner similar to \bar{X} -Chart. The center line of the control chart is the average range, and the upper and lower control limits are computed as follows;

$$CL = \bar{R} = \frac{\sum_{i=1}^k \bar{R}}{k} \quad (2.6)$$

$$UCL = D_4\bar{R} \quad (2.7)$$

$$LCL = D_3\bar{R} \quad (2.8)$$

Whereas the values of D_4 and D_3 are obtained from the table (see appendix). When $D_3 = 0 \forall n \leq 6$. $LCL_R = 0$. Actually, three signal limits often yield a negative lower control limit, which is recorded as zero. This implies that, with samples of six (6) or fewer, it will be impossible for value on the R-Chart to fall outside the lower limit. In effect, the R-Chart will not be capable of detecting reductions in the dispersion of the process output.

2.3. Constructing the P - Chart

The computation of the center line as well as the upper and lower control limits is similar to the computation for other kinds of control charts. CL is computed as the average proportion defective in the population, \bar{P} :

$$\bar{P} = \frac{\text{Total number of Defective Items}}{\text{Total number of Observations}}$$

$$UCL = \bar{P} + Z\sigma_p \quad (2.9)$$

$$LCL = \bar{P} - Z\sigma_p \quad (2.10)$$

Where; Z =Standard normal variable.

\bar{P} =The sample proportion defective.

σ_p =The standard deviation of the average proportion.

As with the other charts, Z is chosen to lie within 3 standard deviation. The sample standard deviation is computed as follows

$$\sigma_p = \sqrt{\frac{\bar{P}(1-\bar{P})}{n}} \quad (2.11)$$

Where n is the sample size.

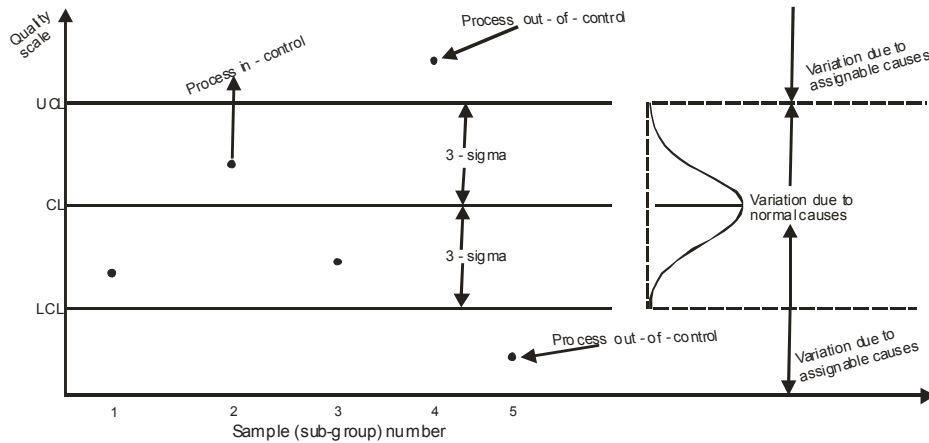


Figure 1. A specimen of a typical control chart.

Where; UCL=Upper Control Limit; CL=Control Limit (center line) LCL=Lower Control Limit.

3. Analysis and Discussion of Results

Computations of the various control limits are presented as follows.

FOR HAZE:

$$\bar{X}_1 = \frac{0.356+0.506+0.375+0.325}{4} = 0.3905$$

$$\bar{X}_2 = \frac{0.297+0.386+0.314+0.325}{4} = 0.3305$$

$$\bar{X}_{20} = \frac{0.792+0.562+0.423+0.55}{4} = 0.5812$$

$$\bar{\bar{X}} = \frac{0.3905+0.3305+\dots+0.5812}{4} =$$

0.4691 (CL: Control limit or Central line)

The control limits:

$$UCL = 0.4691 + 0.729(0.2344) = 0.6398$$

$$LCL = 0.4691 - 0.729(0.2344) = 0.2982$$

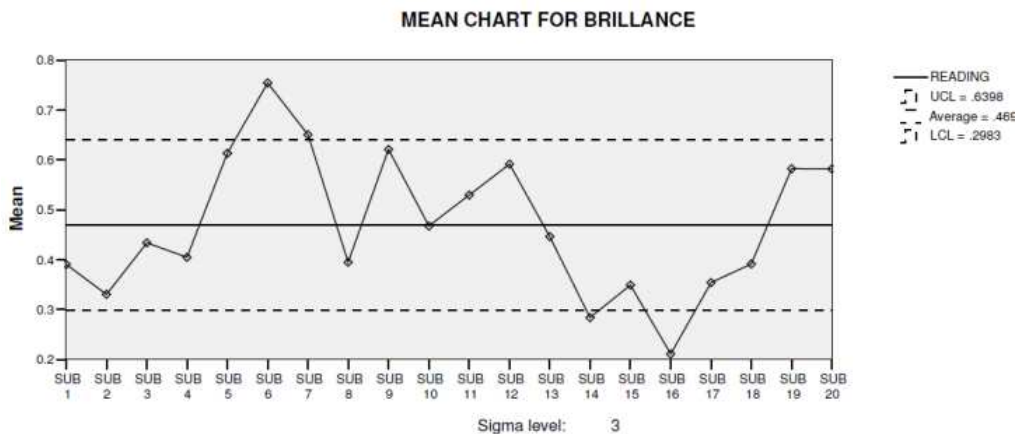


Figure 2. Mean Control Chart for BRILLANCE.

Similarly; for the range,

$$CL = \bar{R} = \frac{0.181 + 0.089 + \dots + 0.369}{20} = 0.2344$$

$$LCL_R = D_3 \bar{R} = 0.00(0.2344) = 0.000$$

$$UCL_R = D_4 \bar{R} = 2.282(0.2344) = 0.5348$$

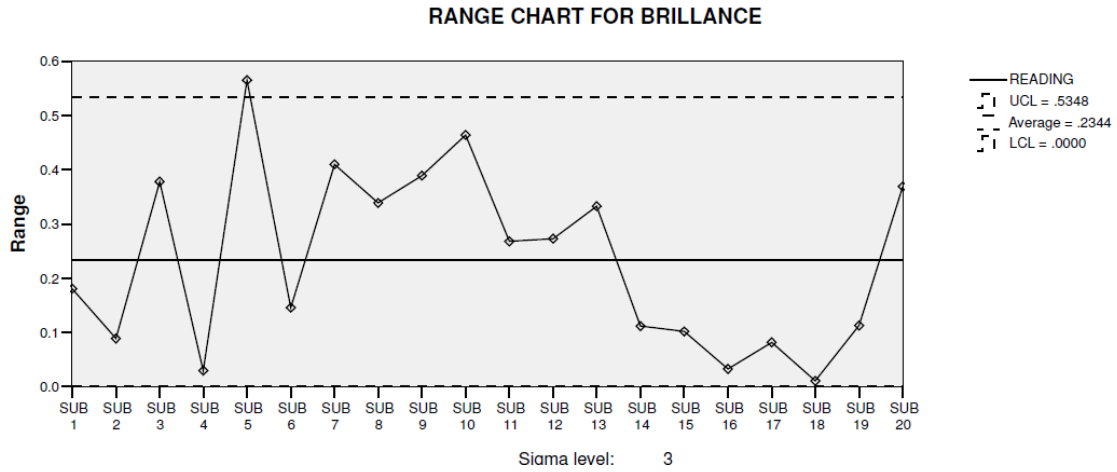


Figure 3. Range Chart for BRILLANCE.

FOR pH)

For Mean chart;

$$\bar{X}_1 = \frac{4.04 + 3.89 + 3.8 + 3.9}{4} = 3.9075$$

$$\bar{X}_{20} = \frac{3.88 + 3.87 + 3.9 + 3.9}{4} = 3.8875$$

$$\bar{\bar{X}} = \frac{3.9075 + \dots + 3.8875}{20} = 3.8854 \text{ (CL: Central line)}$$

The control limits: $UCL = \bar{\bar{X}} + A_2 \bar{R} = 3.8854 + 0.729(0.156) = 3.9990$

$$LCL = \bar{\bar{X}} - A_2 \bar{R} = 3.8854 - 0.729(0.156) = 3.7717$$

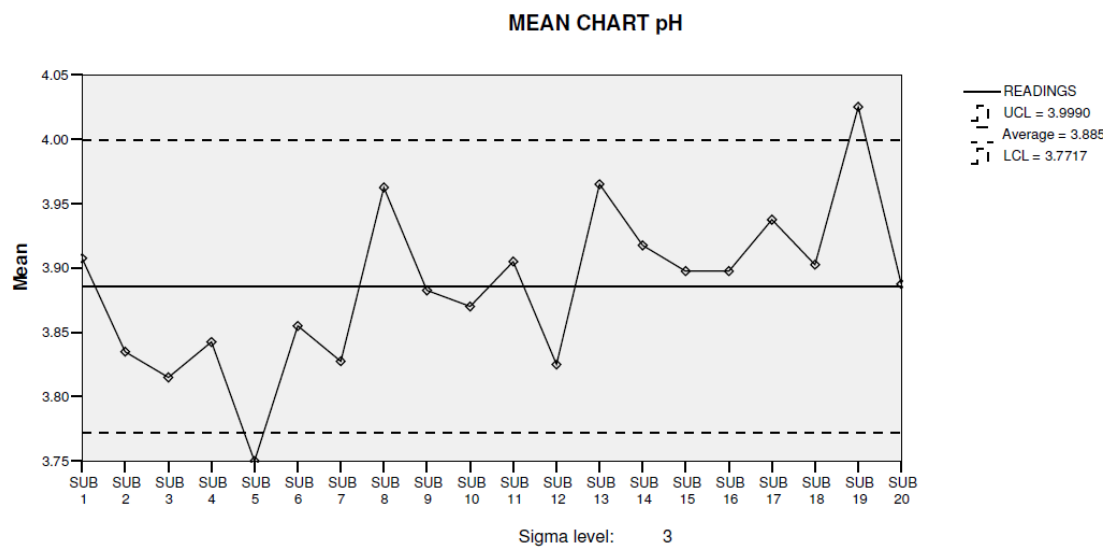


Figure 4. Mean Chart for pH Level.

Similarly, for Range: $CL = \bar{R} = 0.156$ $UCL_R = D_4 \bar{R} = 2.282(0.156) = 0.3560$; $LCL = 0(0.156) = 0.000$

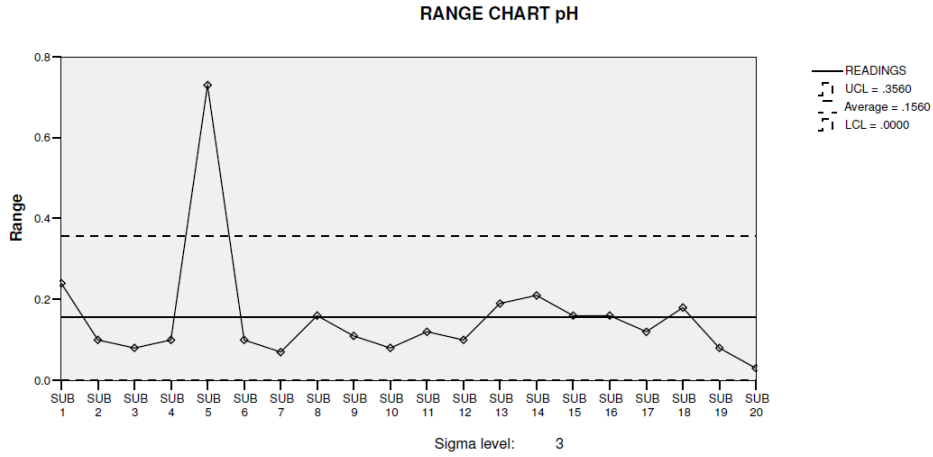


Figure 5. Range Chart for pH Level.

FOR ORIGINAL GRAVITY

For Mean chart:

$$\bar{X}_1 = \frac{11.19+11.12+11.12+11.14}{4} = 11.1425$$

$$\bar{X}_2 = \frac{11.19+11.27+11.18+11.36}{4} = 11.25$$

$$\bar{X}_{20} = \frac{11.12+11.23+11.18+11.28}{4} = 11.2025$$

$$\bar{\bar{X}} = \frac{11.1425+11.25+\dots+11.2025}{4} = 11.2015 \text{ (CL: Central line)}$$

The control limits:

$$UCL = \bar{\bar{X}} + A_2 \bar{R} = 11.2015 + 0.729(0.171) = 11.3262$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R} = 11.2015 - 0.729(0.171) = 11.0768$$

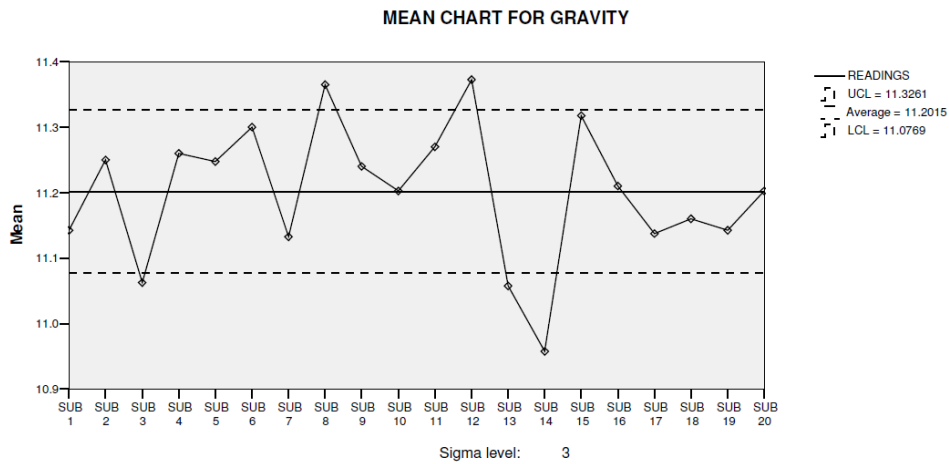


Figure 6. Control Chart for Mean GRAVITY.

For the range:

$$CL = \bar{R} = \frac{0.07+0.18+\dots+0.16}{20} = 0.171; UCL_R = D_4 \bar{R} = 2.282(0.171) = 0.3902$$

$$LCL_R = D_3 \bar{R} = 0.00(0.171) = 0.00$$

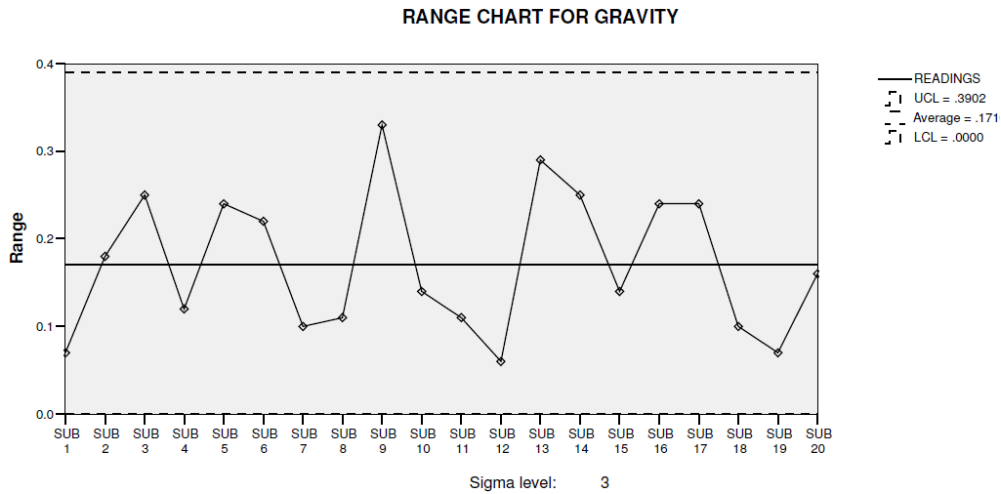


Figure 7. Range Chart for GRAVITY.

(ALCOHOL)

$$\bar{X}_1 = \frac{4.97+4.92+4.91+4.95}{4} = 4.9375$$

$$\bar{X}_2 = \frac{4.8+4.99+4.85+4.81}{4} = 4.8425$$

$$\bar{X}_{20} = \frac{4.9375+4.9975+\dots+4.8425}{4} = 4.6874 \text{ (CL: Central line)}$$

The control limits:

$$UCL = \bar{\bar{X}} + A_2 \bar{R} = 4.6874 + 0.729(0.1205) = 4.7752 \quad LCL = \bar{\bar{X}} - A_2 \bar{R} = 4.6874 - 0.729(0.1205) = 4.5996$$

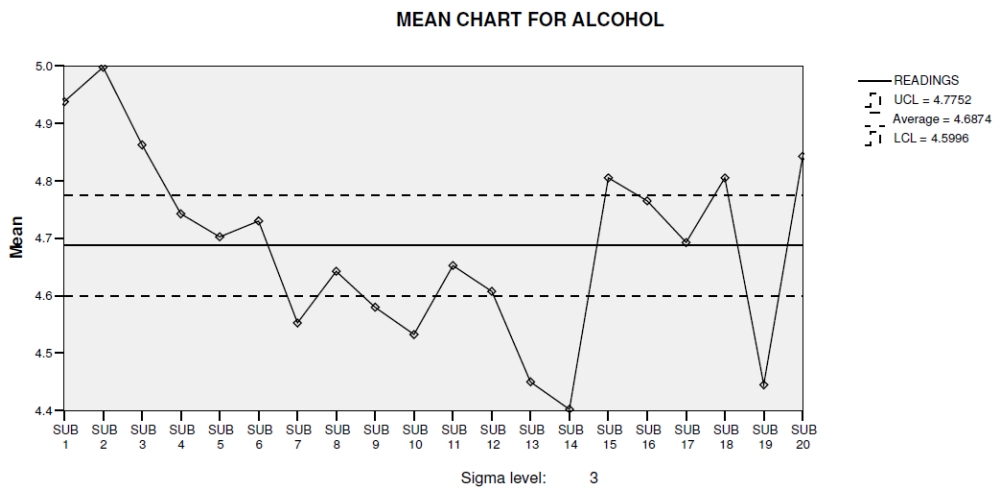


Figure 8. Mean Control Chart for ALCOHOL.

For range:

$$CL = \bar{R} = \frac{0.06+0.08+\dots+0.11}{20} = 0.1205$$

$$UCL_R = D_4 \bar{R} = 2.282(0.1205) = 0.2750$$

$$LCL_R = D_3 \bar{R} = 0(0.1205) = 0.000$$

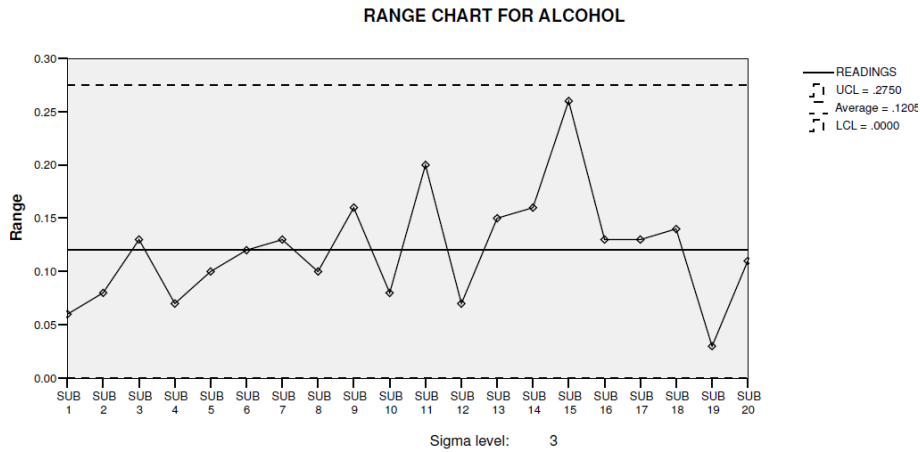


Figure 9. Range Control Chart for ALCOHOL.

P-chart:

$$CL = \bar{P} = \frac{2408}{225000} = 0.0107$$

$$\sigma_p = \sqrt{\frac{\bar{P}(1-\bar{P})}{n}} = \sqrt{\frac{0.0107(1-0.0107)}{15000}} = 0.00084$$

$$UCL = 0.0107 + 3(0.00084) = 0.01322$$

$$LCL = 0.0107 - 3(0.00084) = 0.008182$$

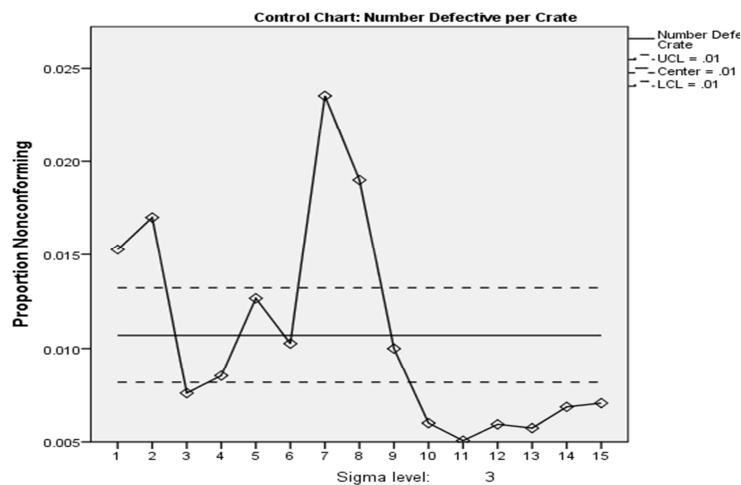


Figure 10. P-Control Chart for Defective Crates per BATCH.

From the plotted points on the mean chart and range chart for BRILIANCE; it could be seen clearly that not all the points plotted are under control, with four (4) points being out - of - control, this may be due to an assignable cause. The points out - of - control were revised and the charts plotted in order to have all the points in control.

For PH level; the mean chart shows that two (2) points are out of control, whereas only 1 point was out of control on the range chart. However the process is not fully in-control until all the out-of-control points are eliminated to have a revised chart for the readings.

For GRAVITY: the mean chart shows that five points were out - of - control, while all the points in the range chart are within control; revising the mean chart may

throw the entire scheme out-of-control, hence total overhaul would be better.

For ALCOHOL level: the mean chart clearly shows that out of 20 points plotted, 11 points are out - of - control; hence the process is totally disregarded because the number of out - of - controls out-numbered the ones in - control. But the range chart did not give any out-of-control point; this is a clear indication that alcohol level needs to be overhauled.

For p-chart, which is meant to access the level of the defects/scraps in their finished products; the chart reflects eleven (11) out-of-control points. This also reveals that the entire production scheme needs total overhauling for the company to maintain optimal productivity level.

4. Summary and Conclusion

In this research four measurable quality characteristics used as raw-materials by a brewing company located in Uyo, in the production of their major products such as beer and Amstel Malta Brilliance (Haze), pH, Original Gravity (O.G) and Alcohol Percentage were obtained. Readings on these characteristics as well as observed defects in their finished product were obtained daily for the period of 15 days. The common statistical process control charts used for variable characteristics, \bar{X} , R charts and for attributes, P-chart were explored. To develop variable control charts for the measurable quality characteristics used during production. Given the initial set objectives of this research, which includes: determining if the process of production is statistically- in - control; building appropriate attributes control chart for the quality of products; and suggesting alternative control schemes for the future in event of out - of - control, we found that the production process based on the observed characteristics, is largely out-of-control across the four quality characteristics. Thus the need for revised control limits, obtained as follows: B (\bar{X} : 0.6563, 0.2738; R : 0.4940, 0.00), pH (\bar{X} : 3.9786, 3.7916; R : 0.2871, 0.00), for future production (See APPENDICES A&B for the appropriate revised charts).

Based on our findings, we therefore recommend as follows:

- Champion Brewery (CB) Plc. should adopt the Statistical Quality method of quality control to improve and maintain the quality of its product.
- CB Plc. should set up a Statistical Quality Control unit and employ trained personnel to carry out routine inspection to ensure consistent and reliable quality characteristics of their end products.
- CB Plc. and other Production Companies should

endeavour to keep accurate statistical data concerning their production so as to help them access improvement on the quality of the subsequent products.

- CB Plc. should revise the control specifications for ORIGINAL GRAVITY and the ALCOHOL PERCENTAGE in their Champion Lager Beer.
- Standard Organisation of Nigeria (SON) should intensify efforts at enforcing quality standards and ensure that all manufacturing industries meet the required standards before being certified to sell their products to consumers.

To conclude, this study has demonstrated importance of statistical quality control in measuring quality performance in manufacturing industries, especially for the companies aiming at producing optimally and at high quality level to the satisfaction of their customers. According to Peters and Waterman (1982), quality is an important element in the pursuit of excellence. It is hoped that those recommendations if given adequate attention would help solve the low quality product problems especially in the Nigerian market; this will ensure products meet consumers' satisfaction and be able to compete with foreign products.

Acknowledgement

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Appendix A

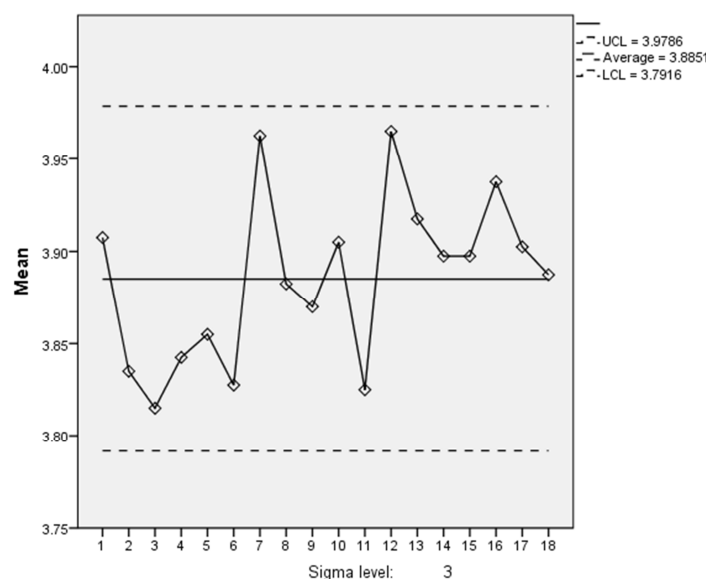


Figure A1. Revised Mean Control Chart for pH.

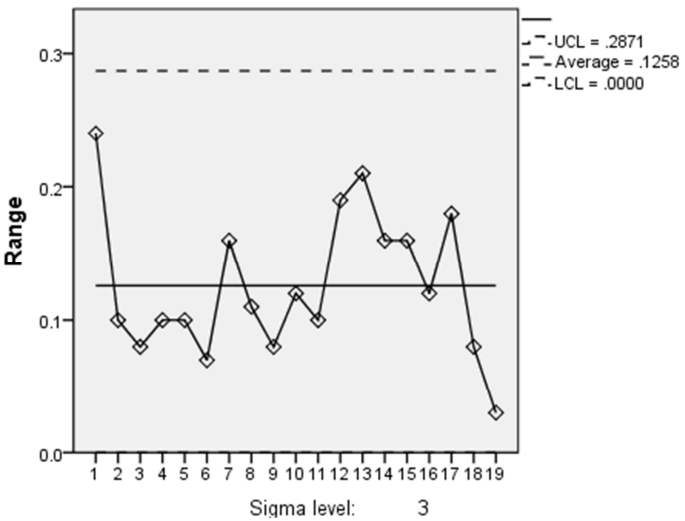


Figure A2. Revised Range Control Chart for pH.

Appendix B

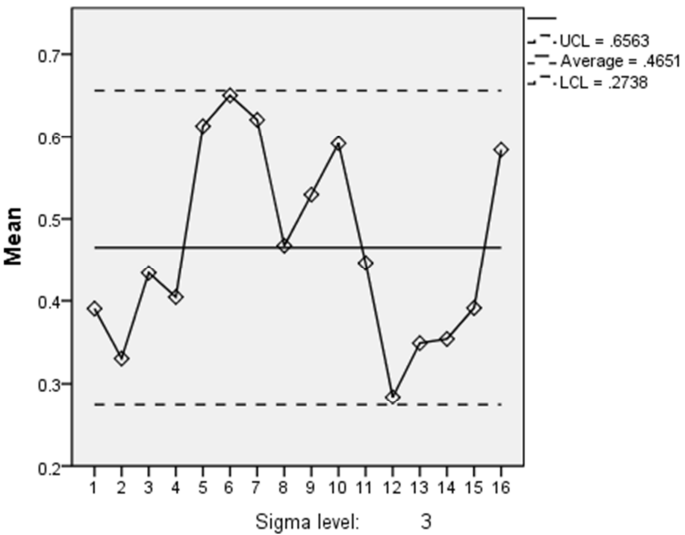


Figure B1. Revised Mean Control Chart for BRILLANCE.

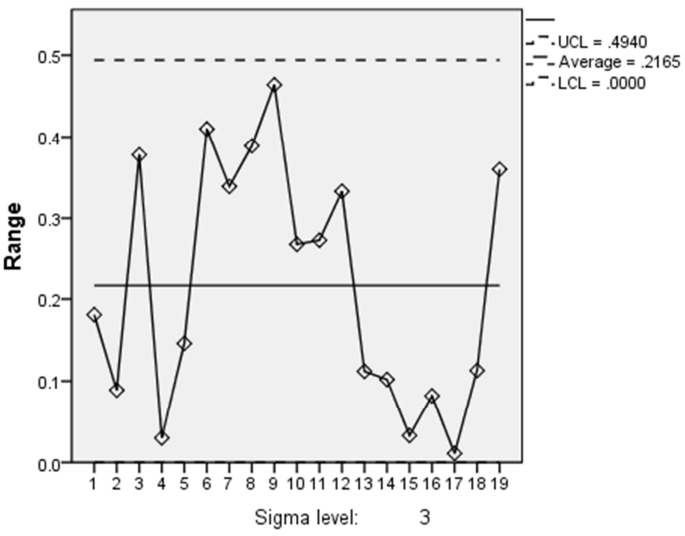


Figure B2. Revised Range Control Chart for BRILLANCE.

Appendix C

Table C1. Control Chart Constants.

Sample size = m	X-bar Chart Constants		for Sigma Estimates	R-Chart Constants		S Chart Constants	
	A_2	A_3	d_2	D_3	D_4	B_3	B_4
2	1.880	2.659	1.128	0	3.267	0	3.267
3	1.023	1.954	1.693	0	2.574	0	2.568
4	0.729	1.628	2.059	0	2.282	0	2.266
5	0.577	1.427	2.326	0	2.114	0	2.089
6	0.483	1.287	2.534	0	2.004	0.030	1.970
7	0.419	1.182	2.704	0.076	1.924	0.118	1.882
8	0.373	1.099	2.847	0.136	1.864	0.185	1.815
9	0.337	1.032	2.970	0.184	1.816	0.239	1.761
10	0.308	0.975	3.078	0.223	1.777	0.284	1.716
11	0.285	0.927	3.173	0.256	1.744	0.321	1.679
12	0.266	0.886	3.258	0.283	1.717	0.354	1.646
13	0.249	0.850	3.336	0.307	1.693	0.382	1.618
14	0.235	0.817	3.407	0.328	1.672	0.406	1.594
15	0.223	0.789	3.472	0.347	1.653	0.428	1.572
16	0.212	0.763	3.532	0.363	1.637	0.448	1.552
17	0.203	0.739	3.588	0.378	1.622	0.466	1.534
18	0.194	0.718	3.640	0.391	1.608	0.482	1.518
19	0.187	0.698	3.689	0.403	1.597	0.497	1.503
20	0.180	0.680	3.735	0.415	1.585	0.510	1.490
21	0.173	0.663	3.778	0.425	1.575	0.523	1.477
22	0.17	0.647	3.819	0.434	1.566	0.534	1.466
23	0.162	0.633	3.858	0.443	1.557	0.545	1.455
24	0.157	0.619	3.895	0.451	1.548	0.555	1.445
25	0.153	0.606	3.931	0.459	1.541	0.565	1.435

Control chart constants for X-bar, R, S, Individuals (called "X" or "I" charts), and MR (Moving Range) Charts.

NOTES: To construct the "X" and "MR" charts (these are companions) we compute the Moving Ranges as:

R_2 = range of 1st and 2nd observations, R_3 = range of 2nd and 3rd observations, R_4 = range of 3rd and 4th observations, etc. with the "average" moving range or "MR-bar" being the average of these ranges with the "sample size" for each of these ranges being $n = 2$ since each is based on consecutive observations... this should provide an estimated standard deviation (needed for the "I" chart) of $\sigma = (\text{MR-bar}) d_2$ where the value of d_2 is based on, as just stated, $m=2$.

Similarly, the UCL and LCL for the MR chart will be: $\text{UCL} = D_4 (\text{MR-bar})$ and $\text{LCL} = D_3 (\text{MR-bar})$

But, since $D_3 = 0$ when $n = 0$ (or, more accurately, is "not applicable") there will be no LCL for the MR chart, just a UCL.

Source: http://web.uni-plovdiv.bg/kiril/Students/Masters-MYK/table_of_control_chart_constants.pdf.

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