Uniform asymptotic solutions of the Cauchy problem for a generalized model equation of L. S. Pontryagin in the case of violation of conditions of asymptotic stability

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Abstract: Here, we construct a uniform asymptotic solution of the Cauchy problem of the small parameter for the inhomogeneous differential equation with small parameter at the derivative, when the linear part of the equation is a pure complex, with its real part changes from negative to positive when one going from the left half to the right half plane.

Keywords: Uniform Asymptotic Solution, the Cauchy Problem, the Small Parameter, Inhomogeneous Differential Equation, Model Equation of L. S. Pontryagin

1. Introduction

In this paper, we consider a linear, inhomogeneous, ordinary differential equation (ODE) of the first order with a small parameter in the derivative in which the equilibrium position when changing the parameters of the system loses its stability. Models with the change of stability occur in various problems of mechanics, electrical engineering, theory of combustion and explosion, biology and other sciences. It was found that in some cases, by changing the stability of the model can be observed phenomenon of delayed loss of stability.

For the first time the phenomenon of delayed loss of stability for singularly perturbed systems has been described by MA Shishkova [1] student academic Pontryagin. Then, many authors have investigated the nature of this phenomenon. Most full effect of delayed loss of stability for singularly perturbed systems has been described by A.I. Neustadt [2]. And there are works S. Karimov, K.S. Alybaev etc. Using the idea of Professor K. Alymkulov [3], we first construct a uniform asymptotic expansion of the solution of the generalized problem more than the previously cited papers.

2. Materials and Methods

Consider the Cauchy problem for a singularly perturbed equation

$$\varepsilon x'(t,\varepsilon) = (t+i)x(t,\varepsilon) + f(t), \tag{1}$$

$$x(-1, \varepsilon) = x^0, \tag{2}$$

where $\varepsilon > 0$ – small parameter, $f(t)$ – analytic function in D, x^0 – is given constant, $t \in D = \{(t_1, t_2): t_1 - (t_2+1) \le 0, t_1 + (t_2+1) \ge 0, -1 \le t_2 \le 0\}$, $x(t)$ – unknown function.

This problem was first considered by a graduate student [1] of academician L.S.Pontryagin and bounded ness are proved of solution on the interval $t \in [-1, 1]$. Here it is constructed a uniform asymptotic expansion of (1)-(2) in $D \supset [-1, 1]$. We will use idea of K.Alymkulov of extract of the singular part for construct asymptotic solution of the singular equation with turning point [3].

The solution of the problem (1)-(2) is sought in the following form

$$x(t,\varepsilon) = \pi_{-1}(\tau,\mu)\mu^{-1} + u_0(t) + \pi_0(\tau,\mu) + (u_1(t) + \pi_1(\tau,\mu))\mu + \ldots + (u_n(t) + \pi_n(\tau,\mu))\mu^n + \ldots \tag{3}$$

here $t+i = \tau\mu$, $\varepsilon = \mu^2$, $u_n(t) \in C^\infty(D)$, $n = 0,1,\ldots$.

Initial date for $\pi_k(\tau,\mu)$, $(k = -1, 0, 1, \ldots)$ we well take next view:

$$\pi_{-1}((-1+i)/\ \mu,\mu)=\mu(x^0-u_0(-1)); \qquad (4)$$

$$\pi_0((-1+i)/\ \mu,\mu)=0; \qquad (5)$$

$$\pi_{2n-1}((-1+i)/\ \mu,\mu)=0,\ n\in N; \qquad (6)$$

$$\pi_{2n}((-1+i)/\ \mu,\mu)=-u_{2n}(-1),\ n\in N; \qquad (7)$$

Inserting (3) in (1) we have:

$$\pi'_{-1}(\tau,\mu)+\mu^2 u_0(t)+\mu\pi'_0(\tau,\mu)+\mu^3 u'_1(t)+\mu^2\pi'_1(\tau,\mu)+\ldots=$$

$$=\tau\pi_{-1}(\tau,\mu)+(t+i)u_0(t)+\mu\tau\pi_0(\tau,\mu)+\mu(t+i)u_1(t)+$$

$$+\mu^2\tau\pi_1(\tau,\mu)+\ldots+f(t)-\sum_{k=0}^{\infty}c_k\varepsilon^k+\sum_{k=0}^{\infty}c_k\mu^{2k},$$

Here we do not exchange this equation the right part of

equality subtracting and adding $\sum_{k=0}^{\infty}c_k\mu^{2k}$. Coefficients c_k we will define below.

We have equation for $u_n(t)$, $\pi_k(\tau,\mu)$:

$$(t+i)u_0(t)+f(t)-c_0=0, \qquad (8)$$

$$\pi'_{-1}=\tau\pi_{-1}+c_0, \qquad (9)$$

$$(t+i)u_1(t)=0, \qquad (10)$$

$$u'_{2(n-1)}(t)=(t+i)u_{2n}(t)-c_n=0, \qquad (11)$$

$$u'_{2n-1}(t)=(t+i)u_{2n+1}(t), \qquad (12)$$

$$\pi'_{2n-1}=\tau\pi_{2n-1}+c_n,\ n\in N. \qquad (13)$$

$$\pi'_0(\tau)=\tau\pi_0(\tau), \qquad (14)$$

$$\pi'_{2n}(\tau)=\tau\pi_{2n}(\tau),\ \ n\in N. \qquad (15)$$

From (8) and (11) correspond receive:
$$u_0(t)=-\frac{f(t)-c_0}{t+i},\ \ \text{let } c_0=f(-i),$$

$$u_{2n}(t)=\frac{u'_{2(n-1)}(t)+c_n}{t+i},\ \ \text{let}\ \ c_n=-u'_{2(n-1)}(-i),$$

then
$$u_0(t)=-(f_1+f_2(t+ia)+f_3(t+ia)^2+\ldots+f_n(t+ia)^{n-1}+\ldots),$$

here $f_k=\dfrac{1}{k!}\dfrac{d^{(k)}f(-i)}{dt^k}$,

$$u_2(t)=-(2f_3+3f_4(t+ia)+4f_5(t+ia)^2+\ldots+(n-1)f_n(t+ia)^{n-3}+\ldots),$$

$$u_4(t)=-(8f_5+15f_6(t+ia)+24f_7(t+ia)^2+\ldots+$$

$$+(n-1)(n-3)f_n(t+ia)^{n-5}+\ldots),$$

$$u_6(t)=-(48f_7+105f_8(t+ia)+192f_9(t+ia)^2+\ldots+$$

$$+(n-1)(n-3)(n-5)f_n(t+ia)^{n-7}+\ldots),$$

$$u_{2n}(t)=-((2n)!!f_{2n+1}+(2n+1)!!f_{2n+2}(t+ia)+\ldots+$$

$$+(k-1)(k-3)\ldots(k-2n+1)f_n(t+ia)^{n-2n-1}+\ldots),\ \ k\geq 2n+1.$$

$$c_1=f_2;\ c_2=3f_4;\ c_3=5*3f_6;\ \ c_4=7*5*3f_8;\ c_n=(2n-1)!!f_{2n}.$$

From (10) and (12) have $u_{2n-1}(t)\equiv 0$, $n\in N$.
Then $u_n(t)\in C^{\infty}(D)$, $n=0,1,\ldots$.
Solutions of problem   (9)-(4), (14)-(5), (13)-(6), (15)-(7) will represent in the view:

$$\pi_{-1}(\tau)=\mu(x^0-u_0(-1))e^{\frac{\mu^2\tau^2+2i}{2\mu^2}}+e^{\frac{\tau^2}{2}}c_0\int_{\frac{-1+i}{\mu}}^{\tau}e^{-\frac{s^2}{2}}ds\ ;$$

$$\pi_{2n-1}(\tau)=e^{\frac{\tau^2}{2}}c_n\int_{\frac{-1+i}{\mu}}^{\tau}e^{-\frac{s^2}{2}}ds\ ,\ n\in N;$$

$$\pi_{2n}(\tau,,\mu)=-u_{2n}(-1)e^{\frac{\mu^2\tau^2+2i}{2\mu^2}}\ ,\ n\in N;$$

$$\pi_0(\tau,\mu)\equiv 0\ .$$

We have

$$\pi_{2n-1}(\tau)=-\frac{c_n}{\tau}\left(1-\frac{1}{\tau^2}+\frac{3}{\tau^4}-\ldots(-1)^k\frac{(2k-1)!!}{\tau^{2k}}+\ldots\right),$$

$$\pi_{2n}(\tau)=0\ ,\ n=0,1,\ldots,\ \ \tau\to\infty\ .$$

Lemma. For the integral

$$J(\tau,\mu)=e^{\tau^2/2}\int_{(-1+i)/\mu}^{\tau}e^{-s^2/2}ds\ ,$$

here $\tau=(t+i)/\mu$.

We have estimate

$$|J(t,\mu)|=\begin{cases}O(\mu),\ if\ t\in D_0\\ O(1),\ if\ t\in D_\mu\end{cases}, \qquad (16)$$

here $t=t_1+it_2$, $D_0=\{(t_1;t_2):t_1+(t_2+1)\geq 0,\ t_1-(t_2+1)\leq-\mu,\ t_2\leq 0\}$,

$D_\mu=\{(t_1;t_2):t^2{}_1-(t_2+1)^2\leq 0,\ -1\leq t_2\leq 0,\ t_1-(t_2+1)\geq-\mu\}$.

Proof.   If   we   turn   to   variable t then

$$J(t,\mu)=e^{\frac{(t+i)^2}{2\mu^2}}\int_{(-1+i)/\mu}^{(t+i)/\mu}e^{-s^2/2}ds=\frac{1}{\mu}e^{\frac{(t+i)^2}{2\mu^2}}\int_{-1}^{t}e^{-\frac{(\xi+i)^2}{2\mu^2}}d\xi\ ,$$

Let $\xi=\xi_1+i\xi_2$, then

$$J(t_1,t_2,\mu) = \frac{1}{\mu} e^{\frac{(t_1+i(t_2+1))^2}{2\mu^2}} \int_L e^{-\frac{(\xi_1+i(\xi_2+1))^2}{2\mu^2}} d(\xi_1+i\xi_2).$$

We consider following cases:

1) If $(t_1,t_2)\in D\cap(t_1\leq 0)$ then way of integrating L consist of segments L_1 and L_2, where L_1: $\xi_1+(\xi_2+1)=0$, $-1\leq\xi_1\leq 0$; L_2: $\xi_1=t_1$, $-1-t_1\leq\xi_2\leq t_2$.

2) If $(t_1,t_2)\in D\cap(t_1\geq 0)$ then way of integrating L consist of segments L_1, L_2 and L_3 where L_1: $\xi_1+(\xi_2+1)=0$, $-1\leq\xi_1\leq 0$; L_2: $\xi_1-(\xi_2+1)=0$, $0\leq\xi_1\leq t_1$; L_3: $\xi_1=t_1$, $t_1-1\leq\xi_2\leq t_2$.

In the first case

$$J(t_1,t_2,\mu)=\frac{1}{\mu}K(t_1,t_2,\varepsilon)\left(\int_{-1}^{t_1}e^{\frac{i\xi_1^2}{\mu^2}}d\xi_1 + \int_{-1-t_1}^{t_2}e^{-\frac{(t_1+i(\xi_2+1))^2}{2\mu^2}}d\xi_2\right), \quad (17)$$

In the second case

$$J(t_1,t_2,\mu)=\frac{1}{\mu}K(t_1,t_2,\varepsilon)\left(\int_{-1}^{0}e^{\frac{i\xi_1^2}{\mu^2}}d\xi_1 + \int_{0}^{t_1}e^{-\frac{i\xi_1^2}{\mu^2}}d\xi_1 + \int_{-1+t_1}^{t_2}e^{-\frac{(t_1+i(\xi_2+1))^2}{2\mu^2}}d\xi_2\right), \quad (18)$$

here $K(t_1,t_2,\varepsilon)=\exp\left(\frac{(t_1+i(t_2+i))^2}{2\mu^2}\right)$.

In (17) integrating by parts we have:

$$\left|\int_{-1}^{t_1}e^{\frac{i\xi_1^2}{\mu^2}}d\xi_1\right|\leq c\mu^2,$$

if $-\delta<t_1\leq 0$, $\delta>0$ – sufficiently small, and by applying the stationary phase method we obtain

$$\left|\int_{-1}^{t_1}e^{\frac{i\xi_1^2}{\mu^2}}d\xi_1\right|\leq c\mu, \quad c>0 - \text{const.}$$

Analogously in (18), we have

$$\left|\int_{-1}^{0}e^{\frac{i\xi_1^2}{\mu^2}}d\xi_1 + \int_{0}^{t_1}e^{-\frac{i\xi_1^2}{\mu^2}}d\xi_1\right|\leq c\mu.$$

If we set $k_1=K(t_1,t_2,\varepsilon)\int_{-1+t_1}^{t_2}e^{-\frac{(t_1+i(\xi_2+1))^2}{2\mu^2}}d\xi_2$, then

$$|k_1|\leq \int_{-1+t_1}^{t_2}e^{-\frac{(t_2-\tau_2)(t_2+\tau_2+2)}{2\mu^2}}d\tau_2 \leq \int_{-1+t_1}^{t_2}e^{-\frac{(t_2-\tau_2)(t_2+t_1+1)}{2\mu^2}}d\tau_2 =$$

$$= \frac{2\mu^2}{1+t_1+t_2}\left(1-e^{\frac{t_1^2-(t_2+1)^2}{2\mu^2}}\right)$$

Thus, we is proved (16).

Consequently, series (3) is asymptotic in domain D.

Series (3) is asymptotical series, that is

$$x(t,\mu)=\frac{\pi_{-1}(\tau)}{\mu}+\sum_{k=0}^{n}\mu^k\left(u_k(t)+\pi_k(\tau,\mu)\right)+ \\ +\mu^{n+1}R_{n+1}(t,\mu),$$

where $|R_{n+1}(t,\mu)|\leq m=const$. Realy for $R_{n+1}(t,\mu)$ we have next problem:

$$\mu^2 R'_{n+1}(t,\mu)=(t+i)R_{n+1}(t,\mu)-\mu u'_n(t),$$

$$R(-1)=0.$$

From here

$$R_{n+1}(t,\mu)=-\frac{1}{\mu}\int_{-1}^{t}u'_n(\tau,\varepsilon)e^{\frac{(t+i)^2-(\tau+i)^2}{2\varepsilon}}d\tau.$$

From lemma 1 follow next estimate

$$|R_{n+1}(t,\mu)|=\begin{cases}O(\mu), & \text{if } t\in D_0,\\ O(1), & \text{if } t\in D_\mu.\end{cases}$$

Theorem. Problem (1)-(2) has a unique solution and satisfies the asymptotic expansion (3).

The expansion (3) can be obtained in another way, i.e. without going into the complex plane.

We write the solution of the problem (1)-(2) in the integral form:

$$x(t,\varepsilon)=x^0 e^{\frac{(t+i)^2-(t_0+i)^2}{2\varepsilon}}+\frac{1}{\varepsilon}\int_{-1}^{t}e^{\frac{(t+i)^2-(\tau+i)^2}{2\varepsilon}}f(\tau)d\tau$$

Let $f(t)=\sum_{k=0}^{\infty}f_k(t+i)^k$, $f_k=\frac{1}{k!}\frac{d^{(k)}f(-i)}{dt^k}$, then

$$x(t,\varepsilon)=x^0 e^{\frac{(t+i)^2-(t_0+i)^2}{2\varepsilon}}+\frac{1}{\varepsilon}\sum_{k=0}^{\infty}f_k I_k(t,\varepsilon),$$

where $I_k(t,\varepsilon)=\int_{-1}^{t}(\tau+i)^k e^{\frac{(t+i)^2-(\tau+i)^2}{2\varepsilon}}d\tau$.

$$I_0(t,\varepsilon)=\int_{-1}^{t}e^{\frac{(t+i)^2-(\tau+i)^2}{2\varepsilon}}d\tau$$

If   differentiate the smal parameter ε, we obtain:

$$I_2(t,\varepsilon)=2\varepsilon^2\frac{\partial I_0(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_0(t,\varepsilon)$$

this implies

$$I_4(t,\varepsilon)=2\varepsilon^2\frac{\partial I_2(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_2(t,\varepsilon);$$

$$I_6(t,\varepsilon)=2\varepsilon^2\frac{\partial I_4(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_4(t,\varepsilon);$$

$$I_{2n}(t,\varepsilon)=2\varepsilon^2\frac{\partial I_{2(n-1)}(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_{2(n-1)}(t,\varepsilon),\ n\in\mathbb{N}.$$

For the odd indices:

$$I_3(t,\varepsilon)=2\varepsilon^2\frac{\partial I_1(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_1(t,\varepsilon);$$

$$I_5(t,\varepsilon)=2\varepsilon^2\frac{\partial I_3(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_3(t,\varepsilon);$$

$$I_{2n+1}(t,\varepsilon)=2\varepsilon^2\frac{\partial I_{2n-1}(t,\varepsilon)}{\partial\varepsilon}+(t+i)^2 I_{2n-1}(t,\varepsilon);$$

$n\in\mathbb{N}$,

where

$$I_1(t,\varepsilon)=\int_{-1}^{t}(t+i)e^{\frac{(t+i)^2-(\tau+i)^2}{2\varepsilon}}d\tau=$$

$$=\varepsilon\left(e^{\frac{(t+i)^2-(t_0+i)^2}{2\varepsilon}}-1\right)=O(\varepsilon).$$

From Lemma for $I_0(t,\varepsilon)$ we obtain:

$$\cdot\left|I_0(t,\mu)\right|=\begin{cases}O(\mu^2),\ \text{при}\ t\in D_0\\ O(\mu),\ \text{при}\ t\in D_\mu\end{cases}$$

Therefore

$$x(t,\varepsilon)=\begin{cases}x^0 e^{\frac{(t+i)^2-(t_0+i)^2}{2\varepsilon}}+\dfrac{1}{\varepsilon}\displaystyle\sum_{k=0}^{\infty}f_k c_k\varepsilon^{(k+1)},\ if\ t\in D_0,\\[2mm] x^0 e^{\frac{(t+i)^2-(t_0+i)^2}{2\varepsilon}}+\dfrac{1}{\varepsilon}\displaystyle\sum_{k=0}^{\infty}f_k c_k\varepsilon^{(k+1)/2},\ if\ t\in D_\mu.\end{cases}$$

it is asymptotically equivalent to the asymptotic series (3).

The integral $I_0(t,\varepsilon)=\int_{-1}^{t}e^{\frac{(t+i)^2-(\tau+i)^2}{2\varepsilon}}d\tau$ can be estimated without having to go into the complex plane as follows.
Introduce a new parameter a.

$$I_0(t,\varepsilon,a)=\int_{-1}^{t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau.$$

and divide the integral into two integrals

$$I_0(t,\varepsilon,a)=\int_{-1}^{t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau=$$

$$=\int_{-1}^{-t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau+\int_{-1}^{-t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau.$$

The first integral is estimated by integration by parts:

$$\int_{-1}^{-t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau=e^{\frac{(t+ia)^2}{2\varepsilon}}\int_{-1}^{-t}e^{-\frac{(\tau+ia)^2}{2\varepsilon}}d\tau=$$

$$=e^{\frac{(t+ia)^2}{2\varepsilon}}\int_{-1}^{-t}\frac{-\varepsilon}{\tau+ia}de^{-\frac{(\tau+ia)^2}{2\varepsilon}}=\varepsilon\left(\frac{1}{ai-1}e^{\frac{(t+ia)^2+2ia^2}{2\varepsilon}}-\right.$$

$$\left.-\frac{1}{(ia-t)}e^{\frac{i2at}{\varepsilon}}-\int_{-1}^{-t}\frac{\varepsilon}{(\tau+ia)^3}de^{-\frac{(\tau+ia)^2}{2\varepsilon}}\right).$$

Therefore

$$\int_{-1}^{-t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau=O(\varepsilon),\ -t<0.$$

We now estimate the second integral:

$$\int_{-t}^{t}e^{\frac{(t+ia)^2-(\tau+ia)^2}{2\varepsilon}}d\tau=e^{\frac{(t+ia)^2}{2\varepsilon}}\int_{-t}^{t}e^{-\frac{(\tau+ia)^2}{2\varepsilon}}d\tau=$$

$$=e^{\frac{(t+ia)^2}{2\varepsilon}}e^{\frac{a^2}{2\varepsilon}}\int_{-t}^{t}e^{-\frac{\tau^2}{2\varepsilon}+i\frac{a\tau}{\varepsilon}}d\tau=$$

$$=e^{\frac{(t+ia)^2}{2\varepsilon}}e^{\frac{a^2}{2\varepsilon}}\int_{-t}^{t}e^{-\frac{\tau^2}{2\varepsilon}}\left(\cos\left(\frac{a\tau}{\varepsilon}\right)-i\sin\left(\frac{a\tau}{\varepsilon}\right)\right)d\tau=$$

$$=e^{\frac{(t+ia)^2}{2\varepsilon}}e^{\frac{a^2}{2\varepsilon}}Y(t,\varepsilon,a);$$

where $Y(t,\varepsilon,a)=\int_{-t}^{t}e^{-\frac{\tau^2}{2\varepsilon}}\cos\left(\frac{a\tau}{\varepsilon}\right)d\tau.$

function $Y(t,\varepsilon,a)$ is differentiable in the parameter a:

$$\frac{\partial Y(t,\varepsilon,a)}{\partial a}=-\int_{-t}^{t}\frac{\tau}{\varepsilon}e^{-\frac{\tau^2}{2\varepsilon}}\sin\left(\frac{a\tau}{\varepsilon}\right)d\tau=$$

$$= e^{-\frac{\tau^2}{2\varepsilon}} \sin\left(\frac{a\tau}{\varepsilon}\right)\Big|_{-t}^{t} - \frac{a}{\varepsilon} \int_{-t}^{t} e^{-\frac{\tau^2}{2\varepsilon}} \cos\left(\frac{a\tau}{\varepsilon}\right) d\tau.$$

As $e^{-\frac{t^2}{2\varepsilon}} \sin(a\tau) \sim 0$, $\varepsilon \to 0$, we have:

$$\frac{\partial Y(t,\varepsilon,a)}{\partial a} = -\frac{a}{\varepsilon} Y(t,a) \Rightarrow Y(t,\varepsilon,a) = ce^{\frac{a^2}{2\varepsilon}}.$$

If $a=0$ then we have $Y(t,\varepsilon,0) = \int_{-t}^{t} e^{-\frac{\tau^2}{2\varepsilon}} d\tau =$

$$= \sqrt{2\varepsilon} \int_{-t/\sqrt{2\varepsilon}}^{t/\sqrt{2\varepsilon}} e^{-z^2} dz \sim \sqrt{2\varepsilon} \int_{-\infty}^{\infty} e^{-z^2} dz = \sqrt{2\varepsilon\pi}.$$

On the other hand $Y(t,\varepsilon,0) = c$.

Therefore $c = \sqrt{2\varepsilon\pi}$ and $Y(t,\varepsilon,a) = \sqrt{2\varepsilon\pi}e^{-\frac{a^2}{2\varepsilon}}$.

$$I_0(t,\varepsilon,a) = O(\varepsilon) + e^{\frac{(t+ia)^2}{2\varepsilon}} e^{\frac{a^2}{2\varepsilon}} \sqrt{2\varepsilon\pi}e^{-\frac{a^2}{2\varepsilon}} =$$

$$= O(\varepsilon) + \sqrt{2\varepsilon\pi}e^{\frac{(t+ia)^2}{2\varepsilon}}.$$

Therefore

$$|I_0(t,\varepsilon)| = O\left(\sqrt{\varepsilon}e^{\frac{t^2-1}{2\varepsilon}}\right).$$

Remark. The considered problem may be generalized to the system of differential equations.

$$\varepsilon x'(t,\varepsilon) = A(t)x(t,\varepsilon) + f(t), \quad x(-1,\varepsilon) = x^0.$$

Where $A(t)$ – matrix-function of the second order have $\lambda_1(t)=t+i$, $\lambda_2(t)=t-i$, eigen values, for example

$$A(t) = \begin{pmatrix} t & -1 \\ 1 & t \end{pmatrix}.$$

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