Review Article

Modeling and Forecasting Kenyan GDP Using Autoregressive Integrated Moving Average (ARIMA) Models

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Abstract: The Gross Domestic Product (GDP) is the market value of all goods and services produced within the borders of a nation in a year. In this paper, Kenya's annual GDP data obtained from the Kenya National Bureau of statistics for the years 1960 to 2012 was studied. Gretl and SPSS 21 statistical softwares were used to build a class of ARIMA (autoregressive integrated moving average) models following the Box-Jenkins method to model the GDP. ARIMA (2, 2, 2) time series model was established as the best for modeling the Kenyan GDP according to the recognition rules and stationary test of time series under the AIC criterion. The results of an in-sample forecast showed that the relative and predicted values were within the range of 5%, and the forecasting effect of this model was relatively adequate and efficient in modeling the annual returns of the Kenyan GDP. Finally, we used the fitted ARIMA model to forecast the GDP of Kenya for the next five years.

Keywords: Gross Domestic Product (GDP), Gretl and SPSS 21 Statistical Softwares, ARIMA (Autoregressive Integrated Moving Average) Models, AIC Criterion

1. Introduction

As an aggregate measure of total economic production for a country, GDP represents the market value of all goods and services produced by the economy during the period measured, including personal consumption, government purchases, private inventories, paid-in construction costs and the foreign trade balance (exports are added, imports are subtracted). It is an area of key interest for most researchers in the field of business in general and of economics in particular. The issue of GDP has become the biggest concern amongst macro economy variables. Data on GDP is regarded as an important index for assessing the national economic development and for judging the operating status of macro economy as a whole [15].

GDP is the aggregate statistic of all economic activity and captures a broader coverage of the economy than other macro-economic variables. It is the market value of all final goods and services produced within the borders of a nation in

a year. It is often considered the best measure of how well the economy is performing. GDP can be measured in three ways. First, the Expenditure approach, it consists of household, business and government purchases of goods and services and net exports. Second, the Production approach, it is equal to the sum of the value added at every stage of production (the intermediate stages) by all industries within the country, plus taxes and fewer subsidies on products in the period. Third is the Income approach, it is equal to the sum of all factor income generated by production in the country (the sum of remuneration of employees, capital income, and gross operating surplus of enterprises i.e. profit, taxes on production and imports less subsidies) in a period [2].

Besides these, it is also a vital basis for government to set up economic developmental strategies and policies. Therefore, an accurate prediction of GDP is necessary to get an insightful idea of future trend of an economy. Raw historical and current data on GDP cannot be used to frame suitable economic development strategies, economic policies and allocation of

funds on different priorities for government as well as individual firms in a particular industry. It needs a reliable estimate of GDP in some period ahead, which is only possible by forecasting GDP as accurately as possible using a suitable time series model. However it is not easy to identify the exact variables that affect the GDP.

2. Literature Review

We provide both theoretical and empirical literature on GDP process and its forecasting. The coverage is organized into three sections. Section 2.1 is on theoretical literature review, section 2.2 is on ARIMA models while section 2.3 is devoted to presentation of empirical literature review; these three sections are further discussed in subsections.

2.1. GDP

Economic growth is measured in terms of an increase in the size of a nation's economy. A broad measure of an economy's size is its output. The most widely-used measure of economic output is the GDP. The three basic ways to determine a nation's GDP are; the Expenditure approach, the Production approach and the Income approach.

The Expenditure Approach of determining GDP adds up the market value of all domestic expenditures made on final goods and services in a single year, including consumption expenditures, investment expenditures, government expenditures, and net exports. Add all of the expenditures together and you determine GDP.

The Production approach, also called the Net Product or Value added method requires three stages of analysis. First gross value of output from all sectors is estimated. Then, intermediate consumption such as cost of materials, supplies and services used in production final output is derived. Then gross output is reduced by intermediate consumption to develop net production.

The Income Approach of determining GDP is to add up all the income earned by households and firms in the year. The total expenditures on all of the final goods and services are also income received as wages, profits, rents, and interest income. GDP is determined by adding together all of the wages, profits, rents, and interest income.

The three methods of measuring GDP should result in the same number, with some possible difference caused by statistical and rounding differences. The credibility of data is always a significant concern in any form of research. An advantage of using the Expenditure Method is data integrity. The source data for expenditure components is considered to be more reliable than for either income or production components.

GDP as examined using the Expenditure Approach is reported as the sum of four components [15]. The formula for determining GDP is:

$$C + I + G + (X - M) = GDP \qquad (1)$$

Where:

C = Personal Consumption Expenditures
I = Gross Private Fixed Investment
G = Government Expenditures and Investment
X = Net Exports
M = Net Imports

2.1.1. Government Expenditure

[16] studied the effect of the size of government expenditure on economic growth for 115 countries for the 1960-1980 periods. He found that although a higher rate of increase in government expense is associated with a higher growth rate a higher share of government expenditure in GDP dampens growth. In his studies, [3] considers government to be complimentary, not a substitute, for private investment, and examines the effect of government expenditures on growth in this light. He found that an increase in government expenditure led to the increase in GDP.

[3] examined an endogenous growth model that suggests a possible relationship between the share of government spending in GDP and the growth rate of per capita real GDP. The key feature of the model by [3] is the presence of constant returns to capital that broadly includes private capital and public services. To the extent that public services are considered an input to production, a possible linkage arises between the size of government and economic growth.

2.1.2. Inflation Rate and GDP

In [10], it is essential to study inflation in each country because inflation is devastating. Inflation created problem and introduced noises in the functioning of the economy that is likely to affect economic growth. However it is not an easy task to tackle the inflation problem effectively. In order to handle inflation problem successfully, accurate assessment of the causes of the problem is critical as strong diagnosis of the nature of the problem will lead to the application of inappropriate cures that might produce unintended adverse effect on the economy.

[12] studied that in the history of inflationary in Malaysia, 1973 and 1974 were exceptional years. Inflation rose significantly in both the international and domestic market in 1973.The sharp oil price increase in 1973 and 1974 was the principal reason for the escalation of world inflation in 1973-1974. However, the effect of an increase in oil price was actually felt in 1974. The substantial price increase in 1973 were bought about the mainly of the shortages of food and raw material arising from bad weather and increased an aggregate demand.

Consequently consumer price in Malaysia began to rise and had reach of high level of 10.62 percent by the end of the year 1973. In 1974, the surge in the oil price by over 230 percent put strong fuel of inflation and the inflation rate in Malaysia was increased to its record high of 17.29 percent. A year later Malaysian economy slumped into its great recession with GDP growth rate of only 0.8 percent in 1975 compared to 8.3 percent in 1974.

The inflation rate in Malaysia was last reported at 2 percent in November of 2010. From 2005 until 2010, the average inflation rate in Malaysia was 2.77 percent reaching an

historical high of 8.50 percent in July of 2008 and a record low of -2.40 percent in July of 2009. Inflation rate refers to a general rise in prices measured against a standard level of purchasing power. The most known measures of Inflation are the CPI which measures consumer prices, and the GDP deflator, which measures inflation in the whole of the domestic economy.

2.1.3. Export

The study by [16] supports the view that export growth promotes overall economic growth. A serious drawback of cross section studies, however, is that the issue of causality between export growth and GDP growth is not address directly. However, faster growing economics may give rise to a greater dynamic export. Many authors have doubted the validity conclusions based on cross country studies. Sheehey (1990), for example investigates whether there are other productive categories besides export whose growth has a similar relationship to GDP. Studies have found that a number of other determinant factors contribute to economic growth.

[1] studied the economic success of new industrial countries such as Indonesia, Malaysia, Philippines, Singapore and Thailand using time data series from the year 1966 until 1998 to find out whether export is the cause of the countries' economic growth. They found that the link between export and economic growth lies in the development policy. Interestingly, their studies also found that it is economic development that causes economic growth, and not vice versa.

Using the approach by [11] of defining GDP net of exports, he found weak support for exports as an engine of growth and very little evidence consistent with a government-led growth hypothesis. [8] found very weak support for the contention that export growth promotes GDP growth. Support for the alternate contention that GDP growth promotes export growth was also weak, although somewhat stronger than the former.

A number of studies have found that export growth exerts a positive impact on GDP growth in less developed countries (LDCs), even when capital and labor are controlled for. Using a similar framework but recognizing the possible heterogeneity of exports, the present paper finds, for the 1960–1980 period, that while the primary export sector exhibits little or no effect on GDP growth in LCDs, there is a differential positive impact by the manufacturing export sector.

Studies by [9] used co-integration analysis and the causality approach by Johansen and ECM to analyze the relationship between consumption expenditure and economic growth. The study concludes that government expenditure may have a role as a catalyst and complement determinant factors to economic growth in Malaysia.

Meanwhile, [18] studied the relationship between per capita saving and per capita GDP in India using the Granger causality test based on the Toda and Yamamoto approach. The data used were from 1950 to 2004. The types of savings include household, corporate and public savings. The results of their studies showed that there are no causal relationships between per capita GDP with per capita household savings or per capita corporate savings coming from any direction.

However, there exists a bilateral causal relationship between per capita household savings and per capita corporate savings.

[22] tried to observe the causal relationship between electricity usage and economic growth amongst four ASEAN countries namely Indonesia, Malaysia, Singapore, and Thailand using modern time series data for the years 1971 to 2002. They found that there is a bilateral causal relationship between electricity use and economic growth in Malaysia and Singapore, while a one-way causal relationship exists towards economic growth through electricity usage in Indonesia and Thailand.

2.1.4. GDP Forecasting

Econometric forecasting involves the application of both statistical and mathematical models to predict future developments in the economy. It allows economists to review past economic trends and forecast how recent economic changes will alter the patterns of past trends.

A time series data of GDP consists of observations generated successively over time. Such data are ordered with respect to time and successive observations may be dependent. The observed time series is generally referred to as time series realization of an underlying process. The data may indicate that there is a trend over time, which is a long term behavior underlying the data. The trend may either be increasing, decreasing, or even constant.

There may be a cyclical fluctuation, which is a pattern of ups and downs over time. Also, the data may show that the underlying process has periodic fluctuations of constant length, which is seasonal behavior. Modeling therefore, captures this underlying process using the observed time series so that one can forecast what would be the likely realization at a time point in future.

In forecasting macroeconomic time series variables like GDP, one has many possible types of models to choose from: vector error correction models, autoregressive conditional heteroskedasticity (ARCH)-based models, or various possible combinations. However, ARIMA models have proven themselves to be relatively robust especially when generating short-run GDP forecasts and have frequently outperformed more sophisticated structural models in terms of short-run forecasting ability [20, 13].

2.2. Auto-regressive Integrated Moving Average (ARIMA) Models

Autoregressive Integrated Moving Average models (ARIMA models) were popularized by George Box and Gwilym Jenkins in the early 1970s. It's an iterative process that involves four stages; identification, estimation, diagnostic checking and forecasting of time series.

According to [5], ARIMA models are a class of linear models that is capable of representing stationary as well as non-stationary. They do not involve independent variables in their construction, but rather make use of the information in the series itself to generate forecasts. ARIMA models therefore, rely heavily on autocorrelation patterns in the data.

ARIMA methodology of forecasting is different from most methods because it does not assume any particular pattern in

the historical data of the series to be forecast. It uses an interactive approach of identifying a possible model from a general class of models. The chosen model is then checked against the historical data to see if it accurately describes the series. Most of the traditional forecasting models therefore, provide a limited number of models relative to the complex behaviour of many time series with little guidelines and statistical tests for verifying the validity of the selected model.

2.2.1. Moving Average (MA) Process

This is a time series model which uses past errors as explanatory variable [19]. Let $u_t(t=1,2,3,...)$ be a white noise process, a sequence of independently and identically distributed (iid) random variables with $E(u_t)=0$ and $Var(u_t) = \sigma^2$. Then the qth order MA model is given as:

$$y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q \quad (2)$$

This model is expressed in terms of past errors and thus we estimate the coefficients $\theta_j, j = 1, ..., q$, and use the model for forecasting. Therefore only q errors will affect the current level y_t but higher order errors do not affect y_t. This implies that it is a short memory model.

2.2.2. Auto-Regression (AR)

According to [22], an autoregressive model of order p, an AR (p) can be expressed as;

$$y_t = c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} \quad (3)$$

Where, $u_t \sim wn(0, \sigma^2)$.

The model is expressed in terms of past values and therefore, we wish to estimate the coefficients $\alpha_j, j = 1, ..., p$, and use the model for forecasting. In this case, all previous values will have cumulative effects on the current level y_t and thus, it is a long-run memory model. The ACF(s) therefore does not die out easily since it takes a longer time to have ACF close to zero.

Partial Autocorrelation Functions (PACF) measures the correlation between an observation k periods ago and the current observation, after controlling for observations at intermediate lags (i.e. all lags <k).

PACF (k) = ACF (k) after controlling the effects of $(y_{t-1}, ..., y_{t-kt1})$. Thus PACF (k) can be found as the coefficient of y_{t-k} in the regression

$$Y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_{k-1} y_{t-k+1} + \alpha_k y_{t-k} + u_t \quad (4)$$

$$\Rightarrow \alpha_k = \text{PACF (K)}$$

Hence the PACF is useful for telling the maximum order of an AR process.

Auto-regressive (AR) models can be coupled with moving average (MA) models to form a general and useful class of time series models called Autoregressive Moving Average (ARMA) models. These can be used when the data are stationary.

2.2.3. Autoregressive Moving Average Model (ARMA)

[21] expressed an ARMA (p, q) model as follows:

$$y_t = c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} \quad (5)$$

This is a combination of both AR and MA models. In this case therefore, neither ACF nor PACF can solely provide the information on the maximum orders of p or q.

This class of models can further be extended to non-stationary series by allowing the differencing of the data series resulting to Autoregressive Integrated Moving Average (ARIMA) models.

2.2.4. Autoregressive Integrated Moving Average (ARIMA) Process

There are a large variety of ARIMA models [4]. The general non-seasonal model is known as ARIMA (p, d, q): where p is the number of autoregressive terms, d is the number of differences and q is the number of moving average terms. A white noise model is classified as ARIMA (0, 0, 0) since there exists no AR part because y_t does not depend on y_{t-1}, there is no differencing involved and also there's no MA part since y_t does not depend on e_{t-1}.

For instance, if y_t is non-stationary, we take a first-difference of y_t so that Δy_t becomes stationary.

$\Delta y_t = y_t - y_{t-1}$ (d = 1 implies one time differencing)

$$\Delta y_t = c + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \cdots + \alpha_p \Delta y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} + u_t \quad (6)$$

is an ARIMA (p, 1, q) model.

A random walk model is classified as ARIMA (0, 1, 0) because there is no AR and MA part involved and only one difference exists.

2.3. Conceptual Framework of Box Jenkins Methodology

According to [5], the process uses four iterative stages of Modeling that involves; identification, estimation, diagnostic checking and forecasting (See figure 1 below).
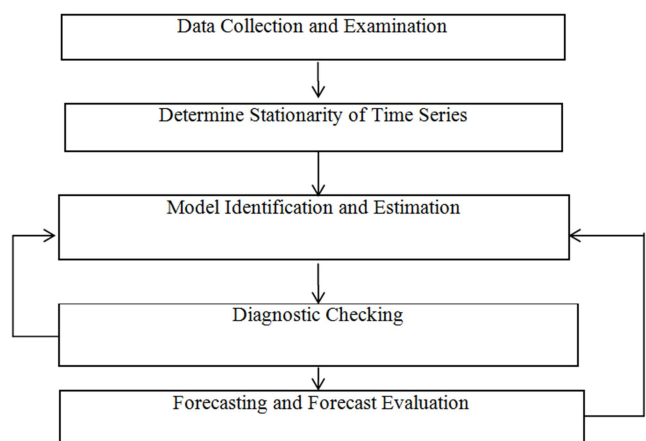

Figure 1. ARIMA forecasting procedure.

2.3.1. Model Identification

A preliminary Box-Jenkins analysis with a plot of the initial data should be run as the starting point in determining an appropriate model. The input data must be adjusted to form a

stationary series and identify seasonality in the dependent series (seasonally differencing it if necessary), and using plots of the autocorrelation and partial autocorrelation functions of the dependent time series to decide which (if any) autoregressive (AR) or moving average (MA) component should be used in the model.

2.3.2. Model Estimation

The parameters of the selected ARIMA (p, d, q) model can be estimated consistently by least-squares or by maximum likelihood. Both estimation procedures are based on the computation of the innovations ε_t from the values of the stationary variable. The least-squares methods minimize the sum of squares;

$$min \sum_t \varepsilon^2 \qquad (7)$$

The log-likelihood can be derived from the joint probability density function of the innovations $\varepsilon_1, \dots \varepsilon_T$, that takes the following form under the normality assumption, $\varepsilon_t \sim N.I.D(0, \sigma^2)$:

$$f(\varepsilon_1, \dots, \varepsilon_T) \propto \sigma^{-T} exp \left\{ -\sum_{t=1}^{T} \varepsilon^2 \big/ 2\sigma^2 \right\} \qquad (8)$$

In order to solve the estimation problem, equations 6 and 7 should be written in terms of the observed data and the set of parameters (Θ, ϕ, σ^2). An ARMA (p, q) process for the stationary transformation $\mathcal{Z}_t$ can be expressed as:

$$\varepsilon_t = \mathcal{Z}_t - \sigma - \sum_{i=1}^{p} \phi_i \mathcal{Z}_{t-1} - \sum_{i=1}^{q} \theta_i \varepsilon_{t-1} \qquad (9)$$

Then, to compute the innovations corresponding to a given set of observations $(\mathcal{Z}_1, \dots, \mathcal{Z}_T)$ and parameters, it is necessary to count with the starting values $\mathcal{Z}_0, \dots, \mathcal{Z}_{p-1}, \varepsilon_0, \dots, \varepsilon_{q-1}$. More realistically, the innovations should be approximated by setting appropriate conditions about the initial values, giving to conditional least squares or conditional maximum likelihood estimators.

2.3.3. Diagnostic Checking

Before using the model for forecasting, it must be checked for adequacy (diagnostic checking). The model is considered adequate if the residuals left over after fitting the model is simply white noise and also the pattern of ACF and PACF of the residuals may suggest how the model can be improved.

Akaike's Information Criterion (AIC) is one of the most robust methods used in estimating parameters of an identified model.

$$AIC = -2\log L + 2m \qquad (10)$$

Where; L denotes the likelihood and m is the number of parameters estimated in the model such that;

$$m = p + q + P + Q \qquad (11)$$

However, not all computer programs produce the AIC or the likelihood L, thus it is not always possible to find the AIC for a given model. A useful approximation to the AIC is therefore denoted as;

$$AIC = n(1 + \log(2\pi) + n \log \sigma^2 + 2m) \qquad (12)$$

As an alternative to AIC, the Bayesian Information Criteria (BIC) and the Schwarz- Bayesian Information Criteria (SBC) are also used as model diagnostics. The SBC is given by;

$$SBC = \log \sigma + (m \log n)/n \qquad (13)$$

2.3.4. Model Forecasting

Model forecasting states the difference between in-sample forecasting and out-of sample forecasting. In-sample forecasting for instance, explains how the chosen model fits the data in a given sample while Out-of-sample forecasting on the other hand, is concerned with determining how a fitted model forecasts future values of the regressand, given the values of the regressors.

To build a reliable model, the following factors are highly considered in forecasting;

a) The level of accuracy required – forecasts should be prepared as accurately as possible to facilitate the decision making process especially made on the basis of the GDP forecasts.

b) Availability of data and information – a wealth of reliable and up-to-date GDP data results to a reliable model.

c) The time horizon that the GDP forecast is intended to cover. This study for instance, covered a short run period.

3. Methodology

3.1. Research Design

The research design was experimental, since the main objective of this study was to determine or forecast the GDP level in Kenya. Experimental research allows the researcher to control the situation and identify the cause and effect relationships between variables and also distinguish placebo effects from treatment effects. According to [12], experimental research is often used where there is time priority in a causal relationship (cause precedes effect), consistency in a causal relationship, and also where the magnitude of the correlation is great.

3.2. Location of the Study

The location of this study was limited to Kenya, a country in East Africa that lies on the equator. With the Indian Ocean to its south-east, it is bordered by Tanzania to the south, Uganda to the west, South Sudan to the north-west, Ethiopia to the north and Somalia to the north-east. Kenya has a land area of 580,000 km^2 and a population of a little over 43 million residents. The country is named after Mount Kenya, a significant landmark and second among Africa's highest mountain peaks. Its capital and largest city is Nairobi.

3.3. Population

According to [14], a target population is the population about which the researcher wishes to study and draw conclusions. In this study, the target population was the Kenya

yearly GDP data from 1960 to 2012. At least more than 50 observations have been identified in order to build a reliable model.

3.4. Data Collection

An extensive time series data is required for univariate time series forecasting. [7] recommends more than 50 observations to build a reliable ARIMA model. In this study, forecasting Kenyan GDP is based on yearly time series data for the period between 1960 and 2012. This implies that the study dealt with GDP time series of Kenya with 53 observations that satisfies the rule of thumb of having more than 50 observations in Box-Jenkins Methodology of time series forecasting.

3.5. Data Analysis

The empirical characteristics of the univariate time series data were checked by obtaining time plots for the data. To gain an insight into univariate processes, autocorrelation and partial autocorrelation functions (ACF and PACF) were considered. The ACF measures the ratio of the covariance between observations k lags apart and the geometric average of the variance of observations (i.e. the variance of the process when it is stationary, as $Var(Y_t) = Var(Y_{t-1})$).

However, some of the observed autocorrelation between Y_t and Y_{t-1} were due to both being correlated with intervening lags. The PACF on the other hand seeks to measure the autocorrelation between Y_t and Y_{t-1} correcting for the correlation with intervening lags.

The log likelihood ratio test, AIC and the BIC were used for model diagnostic checks. Adequacy of the model was carried out for all cases through the analysis of the residuals by use of the Ljung-Pierce Q-statistics. In addition to the residual plots, the Maximum Likelihood Estimate (MSE) was used to check on the efficiency of the model. These were facilitated by use of Gretl statistical software.

Table 1. Data analysis matrix.

Research hypothesis	Independent Variable	Dependent Variable	Statistics
Modeling and forecasting Kenyan GDP	Time	GDP	Time plots, Correlogram, AIC, SBC, Log-likelihood, Cross tabulations, Least Squares, Durbin Watson , Students t

4. Main Results and Discussion

4.1. Basic Analysis

This study used a single set of data for Modeling that comprised of annual levels of GDP for Kenya. The data was obtained from the World Economic Outlook Database and the Kenya National Bureau of Statistics (KNBS) open data from 1960 to 2012. The preliminary analysis of the data was done by use of time plots for the series as shown by Figures 2 and 3 respectively.
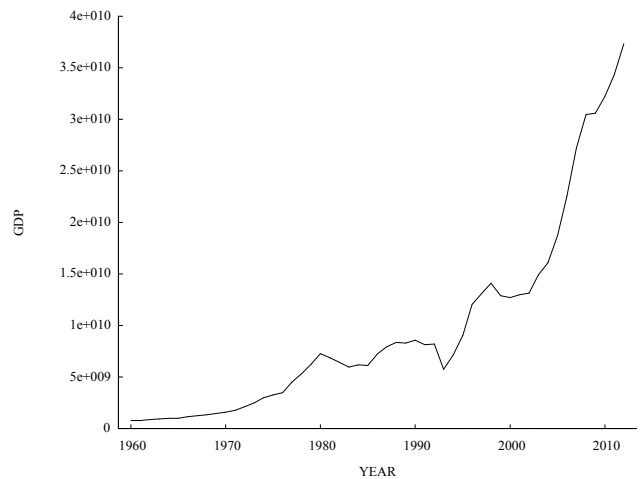

Figure 2. Time plots for annual GDP levels.

From figure 1 above, a visual inspection of the time plots indicates that Kenyan GDP has shown the trend of exponential growth. This implies that both the mean and the variance are not constant. Therefore we regard it as a non-stationary time series.
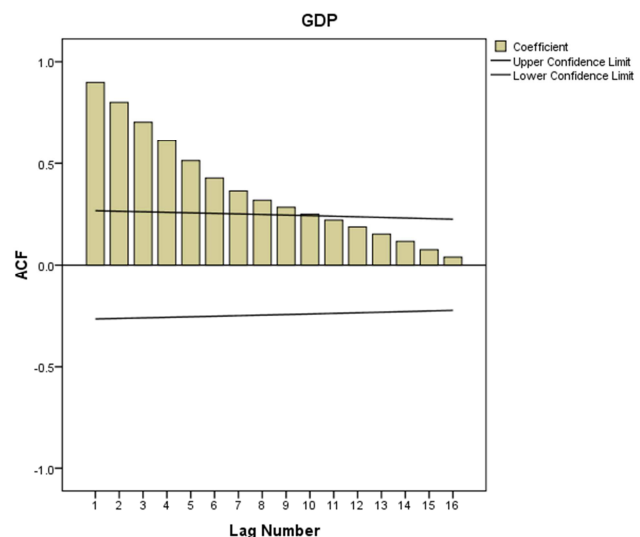

Figure 3. Correlogram for Kenyan GDP data.

A visual examination of the correlogram above confirms that the Kenyan GDP data is non-stationary. This kind of non-stationary time series which contains a seasonal trend can often be carried out by logarithmic transformation. The result is that the exponential trend will be transformed into a linear trend. Before embarking on further analysis using the Box-Jenkins methodology the data has to be transformed to achieve stationarity.

The series was transformed by taking the second differences of the natural logarithms of the values in the series so as to attain stationarity in the second moment. The equation representing the transformation is given by; $Y_t = \ln(P_t) - \ln(P_{t-2})$ where P_t represents the annual values for the series. The time plots for the returns are presented in figure 3.
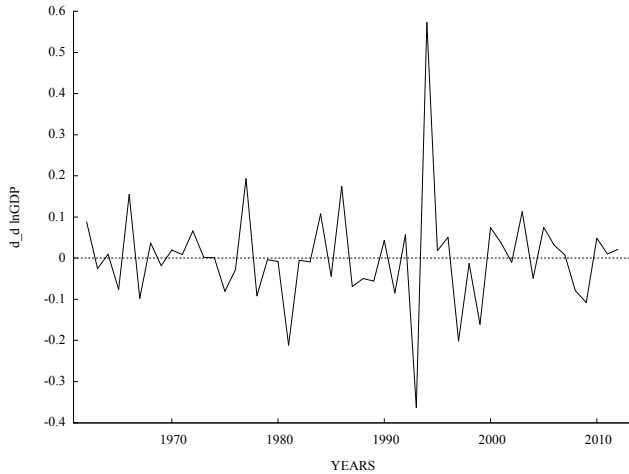
Figure 4. Time plots for log differenced to degree two series.

4.2. Estimation Results

Modeling results of an ARIMA (2, 2, 2) process have been estimated by use of the Gaussian MLE Criterion and are presented in the table 2.

Table 2. Parameter estimates.

Variable	Estimate	Std Error	Z	p-Value
AR(1)	-0.424586	0.228786	-1.856	0.0635
AR(2)	0.395116	0.130549	3.027	0.0025
MA(1)	-0.265205	0.225381	-1.177	0.2393
MA(2)	-0.734795	0.222153	-3.308	0.0009

4.2.1. Interpretation of the Estimation Results

The coefficient estimates of AR (1), AR (2), MA (1) and MA (2) schemes of Kenyan GDP shown in table 4, are statistically significant at 5 percent level of significance. Also, the estimates of AIC, SBC, Log likelihood and the Hannan-Quinn Criterion provide the minimum value hence implying a goodness of fit of the statistical model. Durbin-Watson statistic is near 2 indicating absence of both positive and negative autocorrelation.

4.2.2. Comparison with Other ARIMA Models

The above model was compared with different ARIMA models by use of model selection criteria such as Akaike information criterion, Log likelihood, Hannan-Quinn and Schwarz criterion, but the above model proved to be relatively robust compared to other competing models. The results are presented in table 3.

Table 3. Evaluation of various ARIMA models.

	ARIMA (1, 2, 0)	ARIMA (1, 2, 1)	ARIMA (1, 2, 2)	ARIMA (2, 2, 0)	ARIMA (2, 2, 1)	ARIMA (2, 2, 2)
Log Likelihood	38.0254	38.3674	43.0448	38.0413	38.69815	44.68355
Schwarz Criterion	-68.1871	-64.9393	-70.3623	-64.2870	-61.6690	-69.70792
Alkaline criterion	-72.0507	-70.7347	-78.0896	-70.0825	-69.3963	-79.36710
Hannan-Quinn	-70.5743	-68.5201	-75.1368	-67.8679	-66.4435	-75.67606
SD of Innovations	0.1146	0.1138	0.10079	0.11458	0.11303	0.097601

The fitted ARIMA models were diagnosed using AIC, SBC and the log likelihood ratio test. Parameter estimation for the ARIMA models was done using the Gaussian MLE criterion. The ARIMA models fitted were adequate since the standardized residuals and squared residuals were not significantly correlated as shown by the Ljung-Box Q statistics. In addition, the J-B statistics strongly rejected the null hypothesis of normality in the residuals for all the series.

According to the results and evaluation of different ARIMA models as presented in tables 4 and 5 respectively, the best model can be re-written as follows:

$$Y_t = -0.424586Y_{t-1} + 0.395116Y_{t-2} - 0.265205Y_{t-1}e_{t-1}$$
$$-0.734795Y_{t-2}e_{t-2} + e_t, \ e_t \sim WN(0, \sigma_t^2) \qquad (14)$$

Where; Y_t represents the value of lnGDP.

From equation (14), basing on a 5 percent level of significance, it is clear that the observations are significant at the first lag and also the interaction between observations and the errors are significant at all the lags for the fitted model.

4.3. Out-of-Sample Forecasts

The study emphasized on forecast performance which suggests more focus on minimizing out-of-sample forecast errors than on maximizing in-sample goodness of fit. The approach adopted was therefore one of model mining with the objective of optimizing forecast performance.

The models efficiencies were evaluated using the Mean Squared Errors (MSE). The model that had the minimal MSE was considered the most efficient. However, other statistical properties especially the diagnostics and goodness of fit tests were considered in choosing the most efficient model. The MSE for the various ARMA models are given in table 4.

Table 4. The MSE of various ARMA Models.

MODEL	MSE
ARIMA (0, 2, 0)	0.015648
ARIMA (0, 2, 1)	0.011664
ARIMA (0, 2, 2)	0.010904
ARIMA (1, 2, 0)	0.01316
ARIMA (1, 2, 1)	0.012972
ARIMA (1, 2, 2)	0.010607
ARIMA (2, 2, 0)	0.013152
ARIMA (2, 2, 1)	0.012804
ARIMA (2, 2, 2)	0.0099453

Considering the MSE values in Table 4 above, it is clear that ARIMA (2, 2, 2) model has the smallest value of the MSE thus the most efficient in Modeling and forecasting the Kenyan GDP. The chosen model therefore, is justified by its relatively lower values of residual Kurtosis and MSE in addition to the other diagnostics considered.

Therefore, other than within sample forecasts presented in appendix 1, the study also estimated five years out-of sample forecasts of the model to measure the forecasting ability. Results indicate that Kenyan GDP will continue to rise.

The forecasting power of the model is very high as indicated by the small difference between Actual and fitted values as presented in appendix 2. The five years ahead forecasts of Kenyan GDP are presented in table 5.

Table 5. Five Years ahead GDP Forecasts.

Year	lnGDP Forecast	Std.Error	GDP Forecast
2013	24.415792	0.097601	40146134650
2014	24.493442	0.160869	43387707470
2015	24.564373	0.222413	46577014250
2016	24.640186	0.270886	50245458410
2017	24.711272	0.316843	53947220100

5. Summary, Conclusion and Recommendations

5.1. Summary

The aim of the study was to model and forecast Kenyan GDP based on Box-Jenkins methodology and providing five years inflation forecasts of Kenya. Through collection and examination of the annual GDP data of Kenya, determining the order of integration, model identification, diagnostic checking, model stability testing, and forecast performance evaluation, the best ARIMA model was proposed in equation (14) based on the least mean squared error criteria. Time plots and the correlogram were used for testing stationarity of the data. Also, the Gaussian MLE Criterion was used for estimating the model.

5.2. Main Findings

The first main empirical finding of the study is the model that has been identified for forecasting GDP and it is presented below:

$$Y_t = -0.424586Y_{t-1} + 0.395116Y_{t-2} - 0.265205Y_{t-1}e_{t-1}$$
$$-0.734795Y_{t-2}e_{t-2} + e_t, e_t \sim WN(0, \sigma_t^2). \qquad (15)$$

Where: Y_t represents the value of lnGDP.

This is the forecasting model of GDP in Kenya that is recommended for consistent forecasting. All coefficients were statistically significant at 5 percent. Other statistical properties especially the diagnostics and goodness of fit tests were considered in choosing the most efficient model. Model efficiency was determined using the Mean Squared Error as shown in table 4.

Various ARIMA models with different order of Autoregressive and Moving Average terms were compared based on their performance, checked and verified by using the statistics such as AIC, SBC, Log-likelihood, Hannan Quinn Criterion and the Jarque-Bera statistic. The results indicate that the proposed model performed well in terms of both in-sample and out-of-sample.

The second empirical finding of the study is the 5 years GDP forecasts of Kenya. The out of sample short-run forecasts obtained indicate an increase in Kenyan GDP level.

5.3. Conclusion and Recommendation

Through time series analysis of Kenyan GDP in the years 1960 to 2007, the ARIMA (2, 2, 2) model was established. Transformation of the series by the model parameters turned the residual sequence into white noise sequence. The fitting result of the model is convincing and practical by using Gretl. The GDP of Kenya is forecasted by using the model.

The result shows that the relative error is within the range of 5%, which is relatively ideal. According to the values predicted, Kenyan GDP shows a higher growth trend in the next five years from 2013 to2017. However, the forecasting result of this model is only a predicted value; the national economy is a complex and dynamic system. The adjustments of macro policy and the changes of the development environment will cause the relative change of macro-economic indicators. Therefore, we should pay attention to the risk of adjustment in the economic operation and maintain the stability and continuity of the microeconomic regulation and control too prevent the economy from severe fluctuations and adjust the corresponding target value according to the actual situation.

5.4. Suggestions for Further Research

From the findings of the study, the following areas are suggested for further research:
i. Analysis of GDP Dynamics in Kenya using different models.
ii. Examination of individual components of the GDP.

Appendices

Appendix 1
Standard error of residuals = 0.0976013

Table 6. In-Sample GDP Forecasts 1960-2012.

Year	Actual GDP	Actual lnGDP	Fitted lnGDP
1962	868111400	20.5818	20.4934
1963	926589400	20.6470	20.6322
1964	998759400	20.7220	20.7166
1965	997919300	20.7212	20.7738
1966	1164520000	20.8756	20.7596
1967	1232559000	20.9324	20.9512
1968	1353295000	21.0258	21.0218
1969	1458379000	21.1006	21.0767
1970	1603447000	21.1954	21.1911
1971	1778391000	21.2990	21.2577
1972	2107279000	21.4687	21.3961
1973	2502142000	21.6404	21.5719
1974	2973309000	21.8129	21.7746
1975	3259345000	21.9048	21.9313
1976	3474542000	21.9687	22.0103
1977	4494379000	22.2261	22.0396
1978	5303735000	22.3917	22.3797
1979	6234391000	22.5533	22.5405
1980	7265315000	22.7064	22.6693

| Year	Actual GDP	Actual lnGDP	Fitted lnGDP
1981	6854492000	22.6482	22.8444
1982	6431579000	22.5845	22.6936
1983	5979198000	22.5116	22.6018
1984	6191437000	22.5464	22.5374
1985	6135034000	22.5373	22.5934
1986	7239127000	22.7028	22.5958
1987	7970821000	22.7991	22.7919
1988	8355381000	22.8462	22.9164
1989	8283114000	22.8375	22.8980
1990	8572359000	22.8718	22.8970
1991	8151489000	22.8215	22.9147
1992	8209121000	22.8285	22.8638
1993	5751786000	22.4728	22.8525
1994	7148149000	22.6901	22.4086
1995	9046320000	22.9256	22.7250
1996	12045860000	23.2120	23.1311
1997	13115760000	23.2971	23.3212
1998	14094000000	23.3690	23.4357
1999	12896010000	23.2802	23.4003
2000	12705350000	23.2653	23.3310
2001	12985990000	23.2871	23.2574
2002	13147740000	23.2995	23.3626
2003	14904500000	23.4249	23.3244
2004	16095320000	23.5018	23.5195
2005	18737900000	23.6538	23.5760
2006	22504140000	23.8370	23.7485
2007	27236740000	24.0278	23.9590
2008	30465490000	24.1399	24.1472
2009	30580370000	24.1436	24.2407
2010	32230612377	24.1962	24.1913
2011	34329924186	24.2593	24.2540
2012	37338072592	24.3433	24.3324

Appendix 2
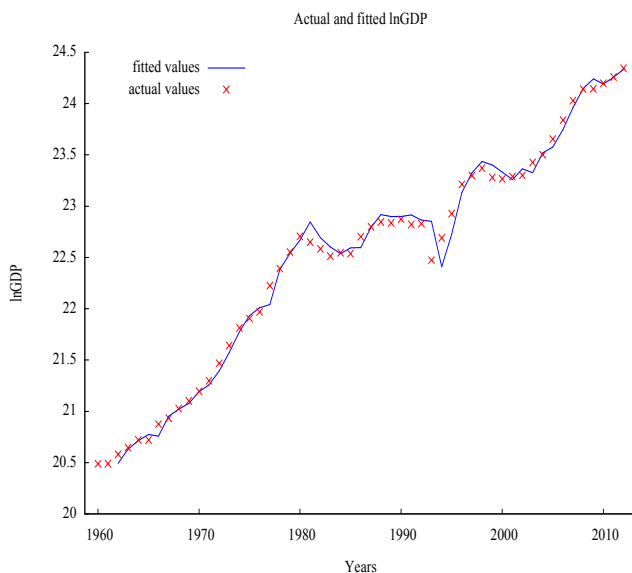
Figure 5. *Time Plot for Actual and Fitted lnGDP values.*

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